LINFO 1121

DATA STRUCTURES AND ALGORITHMS

UCLouvain

TP3: Arbres de recherche

Question 3.1.1 Sequential vs BINARY SEARCH

	Sequential	Binary
Insertion	O(n)	O(n) O(n/2) moyen
Recherche	O(n)	O(log n)

n : nombre d'éléments dans l'array

Question 3.1.1 Sequential vs BINARY SEARCH

	Sequential	Binary
Insertion	O(n)	O(n) O(n/2) moyen
Recherche	O(n)	O(log n)

seq =

bin =

P : Nombre de put

G : Nombre de get

Question 3.1.1 Sequential vs BINARY SEARCH

Coûts liés aux puts

$$\text{seq} = (\sum_{i=1}^{P} i)$$

$$\text{bin} = (\sum_{i=1}^{P} i)$$

	Sequential	Binary
Insertion	O(n)	O(n) O(n/2) moyen
Recherche	O(n)	O(log n)

P : Nombre de put

G : Nombre de get

Question 3.1.1 Sequential vs BINARY SEARCH

Coûts liés aux puts

+ coûts liés aux gets

$$\text{seq} = (\sum_{i=1}^{P} i) + GP$$

$$\text{bin} = (\sum_{i=1}^{P} i) + G \log P$$

	Sequential	Binary
Insertion	O(n)	O(n) O(n/2) moyen
Recherche	O(n)	O(log n)

P : Nombre de put

G : Nombre de get

Question 3.1.1 Sequential vs BINARY SEARCH

$$\text{seq} = \left(\sum_{i=1}^{P} i\right) + GP \qquad \text{bin} = \left(\sum_{i=1}^{P} i\right) + G \log P$$

$$GP > G \log P$$

$$\text{seq} > \text{bin}$$

Rappels sur les BSTS

Propriété d'un Binary Search Tree

C'est un arbre :

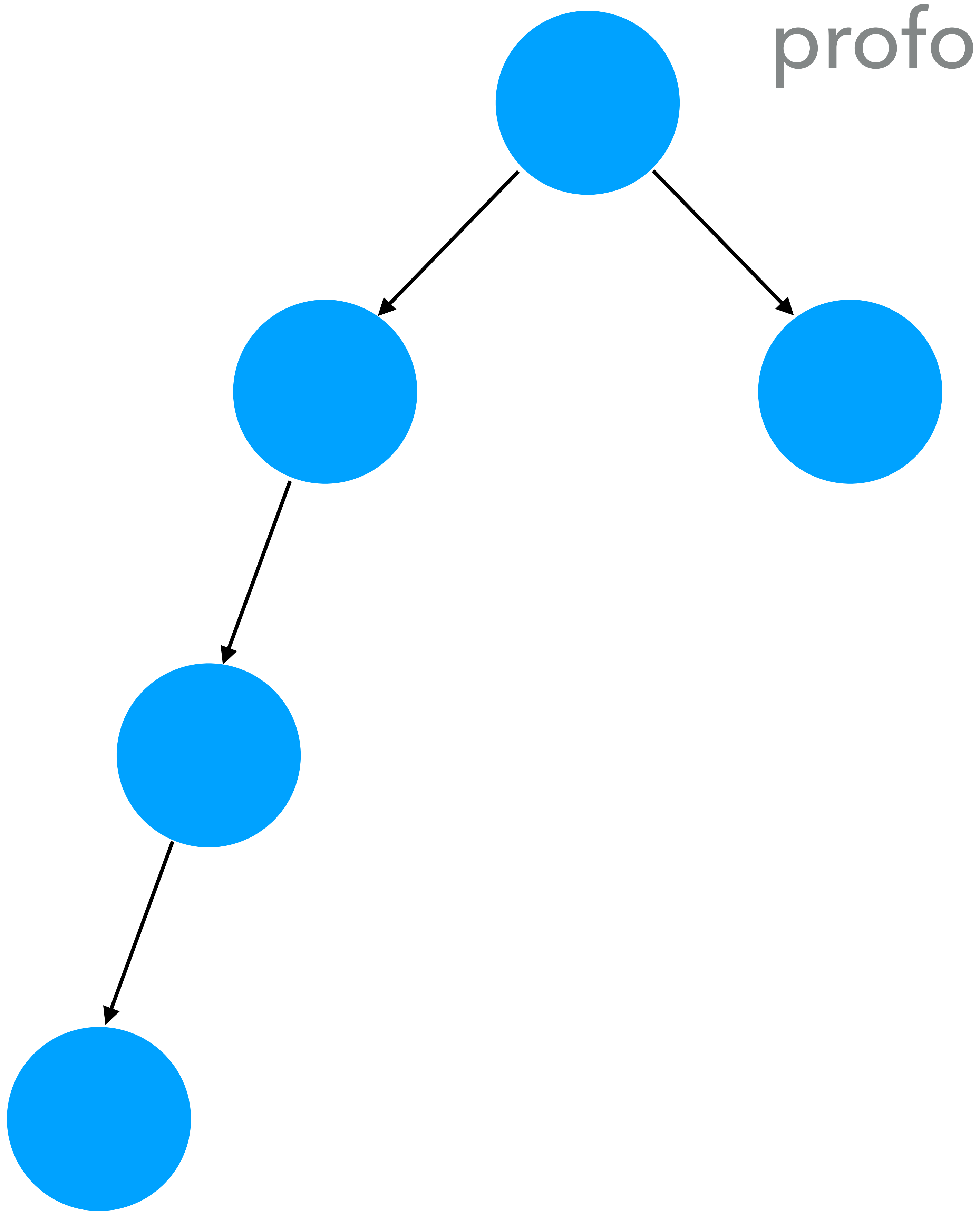
- C'est un graphe
- Connexe
- Possédant n-1 arêtes pour n noeuds
- Pas de cycles
- Au plus un chemin entre toute paire de noeuds

Il est binaire :

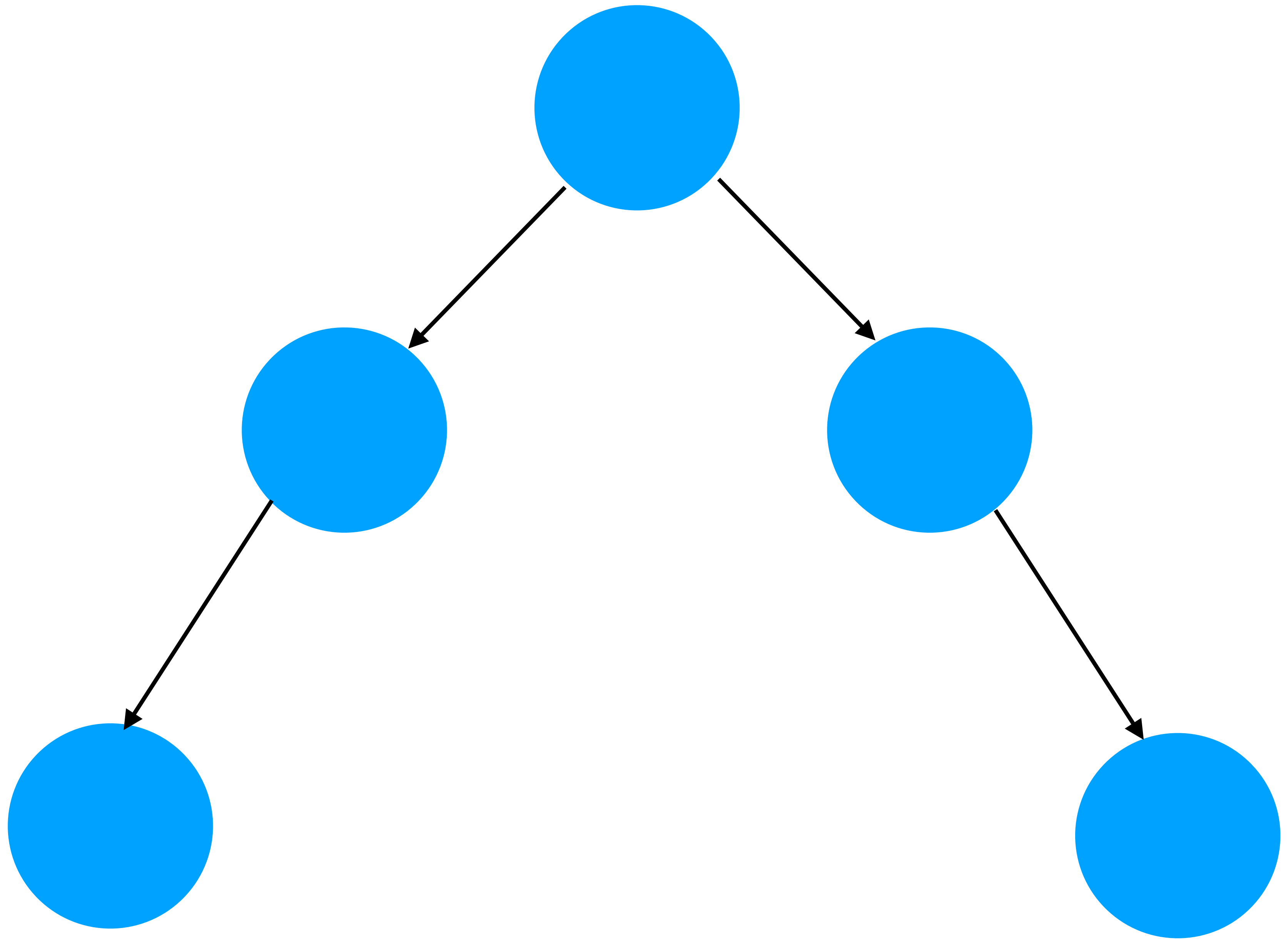
- Deux enfants par noeuds maximum
- Les noeuds en dessous, et à **gauche** doivent avoir des clés plus **petites**
- Les noeuds en dessous, et à **droite** doivent avoir des clés plus **grandes**

Équilibre d’un arbre

Équilibré : toute les feuilles sont à la même profondeur (+/- 1 au maximum)

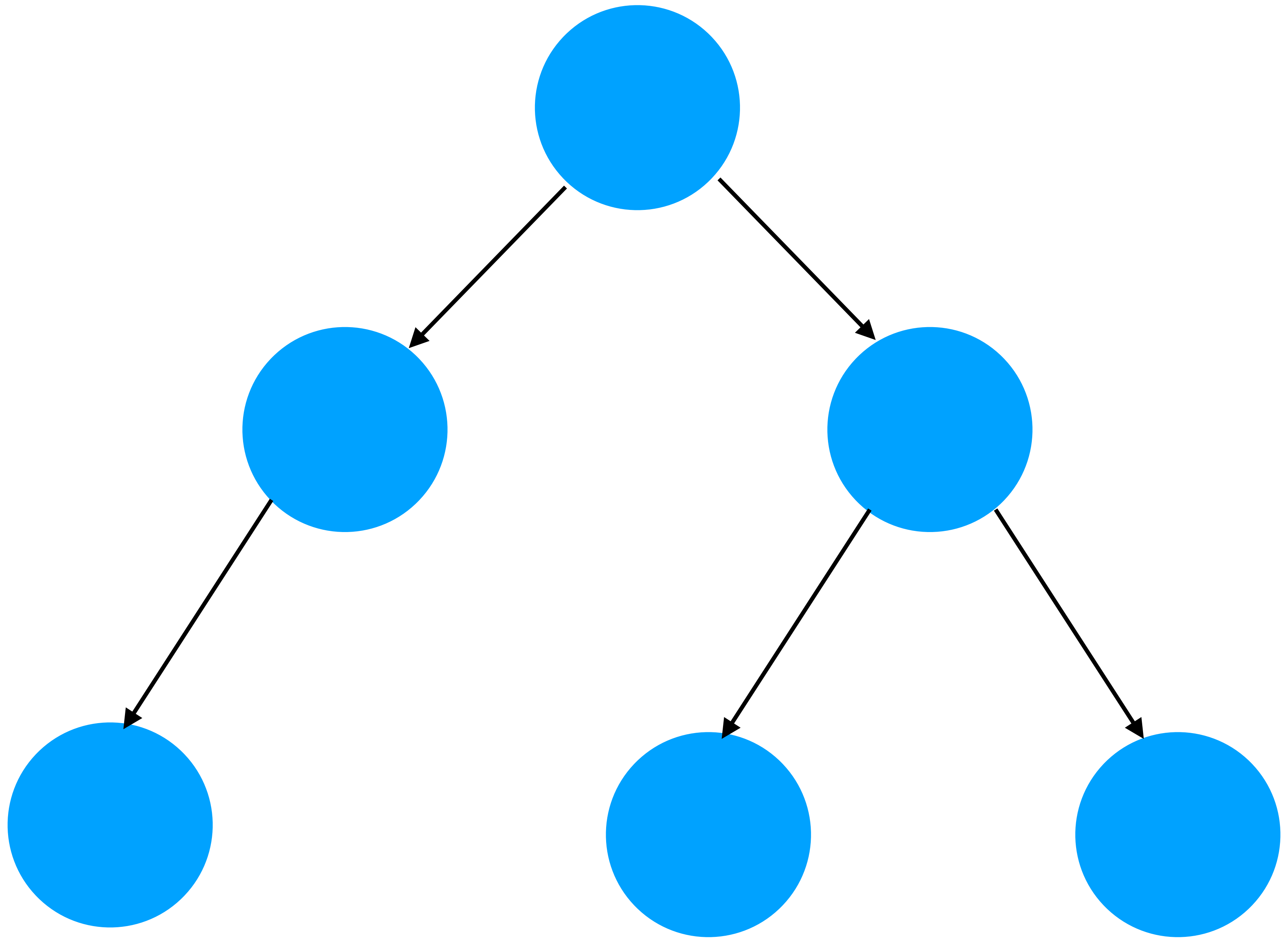


Non équilibré



Équilibré

Équilibre parfait d'un arbre



Parfaitement équilibré si taille des ces chemins est de $\log_2(n)(+/-\ 1)$.

3.1.3 Interpolation Search

```
public int rank(Key key) {
        Key lo = min(Keys)
        Key hi = max(Keys)
        while (lo <= hi) {
            int index = floor((key - lo)/(hi - lo));
            int cmp = key.compareTo(keys[index]);
            if      (cmp < 0) hi = index - 1;
            else if (cmp > 0) lo = index + 1;
            else return mid;
        }
        return lo;
    }
```

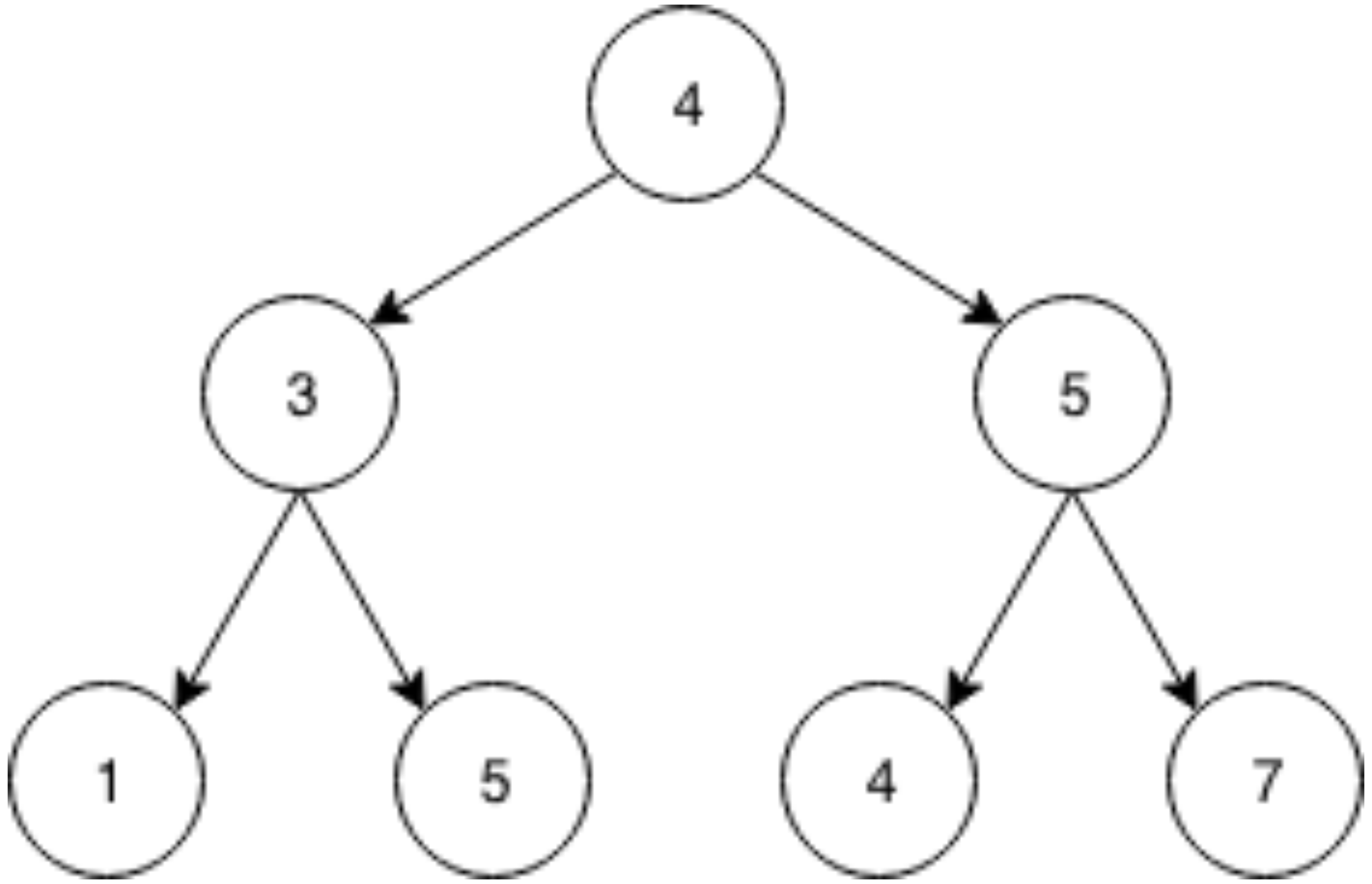
3.1.4 *Caching* key

```
public class BinarySearchST <Key extends Comparable<Key>, Value> {
    private Key[] keys;
    private Value[] vals;
    private int N;
    private Key cacheKey;
    private int cacheRank;
```

```
public int rank(Key key) {
    if (key.compareTo(cacheKey) == 0)
        return cacheRank;

    int lo = 0, hi = N-1;
    while (lo <= hi) {
        int mid = lo + (hi - lo) / 2;
        int cmp = key.compareTo(keys[mid]);
        if      (cmp < 0) hi = mid - 1;
        else if (cmp > 0) lo = mid + 1;
        else {
            cacheKey = key;
            cacheRank = mid;
            return mid;
        }
    }
    return lo;
}
```

Question 3.1.5 isBST()

```
private boolean [illegible]ST(Node x) {
    if (x == null) [illegible] true;
    // Est-ce que l'e[illegible]ien une plus petite clé ?
    if (x.key.compare[illegible]= 0) return false;
    // Est-ce que l'enf[illegible]bien une plus grande clé ?
    if (x.key.comp[illegible]0) return false;
    return isBST[illegible] && is[illegible]ight);
}
```



Question 3.1.5 isBST()

Ajout de bornes supérieures et inférieures

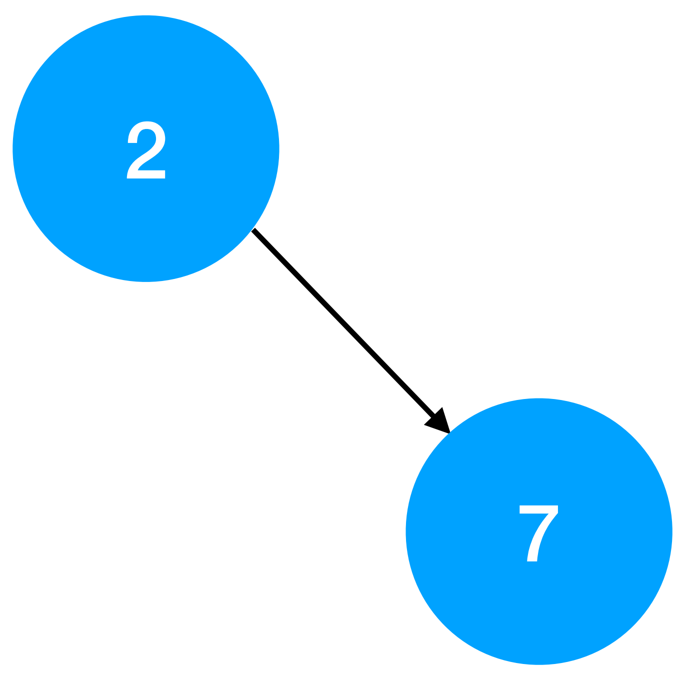
```
private boolean isBST() {
    return isBST(root, null, null);
}

private boolean isBST(Node x, Key min, Key max) {
    if (x == null) return true;
    if (min != null && x.key.compareTo(min) <= 0) return false;
    if (max != null && x.key.compareTo(max) >= 0) return false;
    return isBST(x.left, min, x.key) && isBST(x.right, x.key, max);
}
```

Question 3.1.6 VISITE POSSIBLES EN DFS

- 10,9,8,7,6,5
- 4,10,8,6,5
- 1,10,2,9,3,8,4,7,6,5
- 2,7,3,8,4,5
- 1,2,10,4,8,5

Question 3.1.6 VISITE POSSIBLES EN DFS

2,7,3,8,4,5

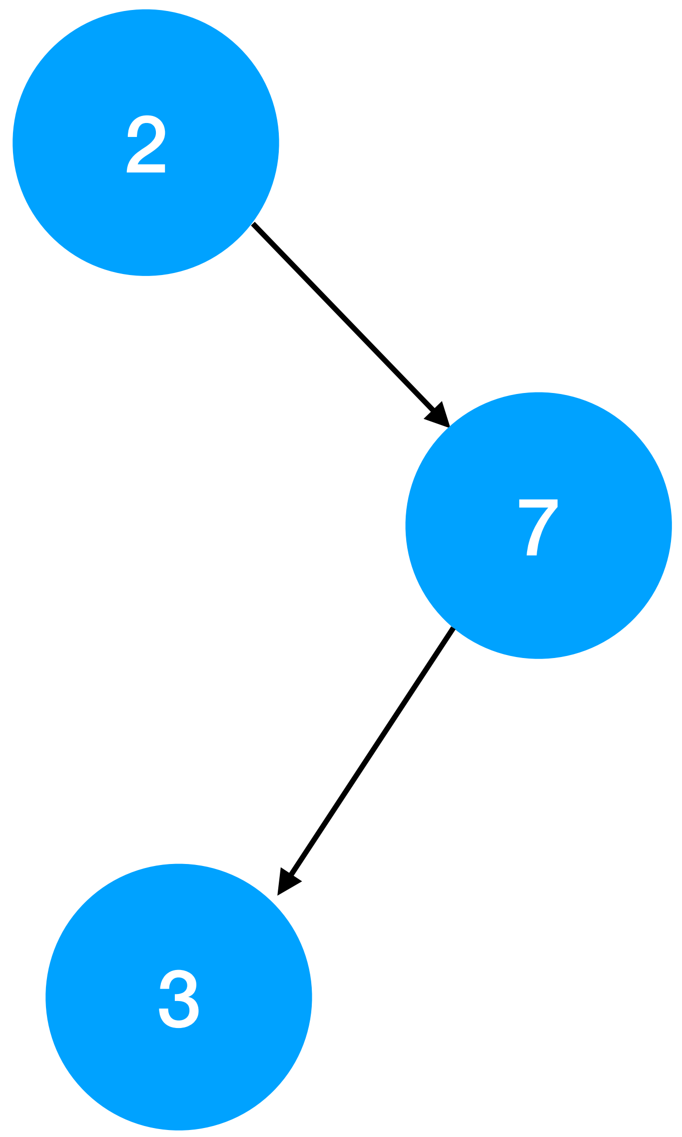


$$LB = -\infty$$
$$UB = +\infty$$

$$7 > 2 \rightarrow LB = 2$$

Question 3.1.6 VISITE POSSIBLES EN DFS

2,7,3,8,4,5



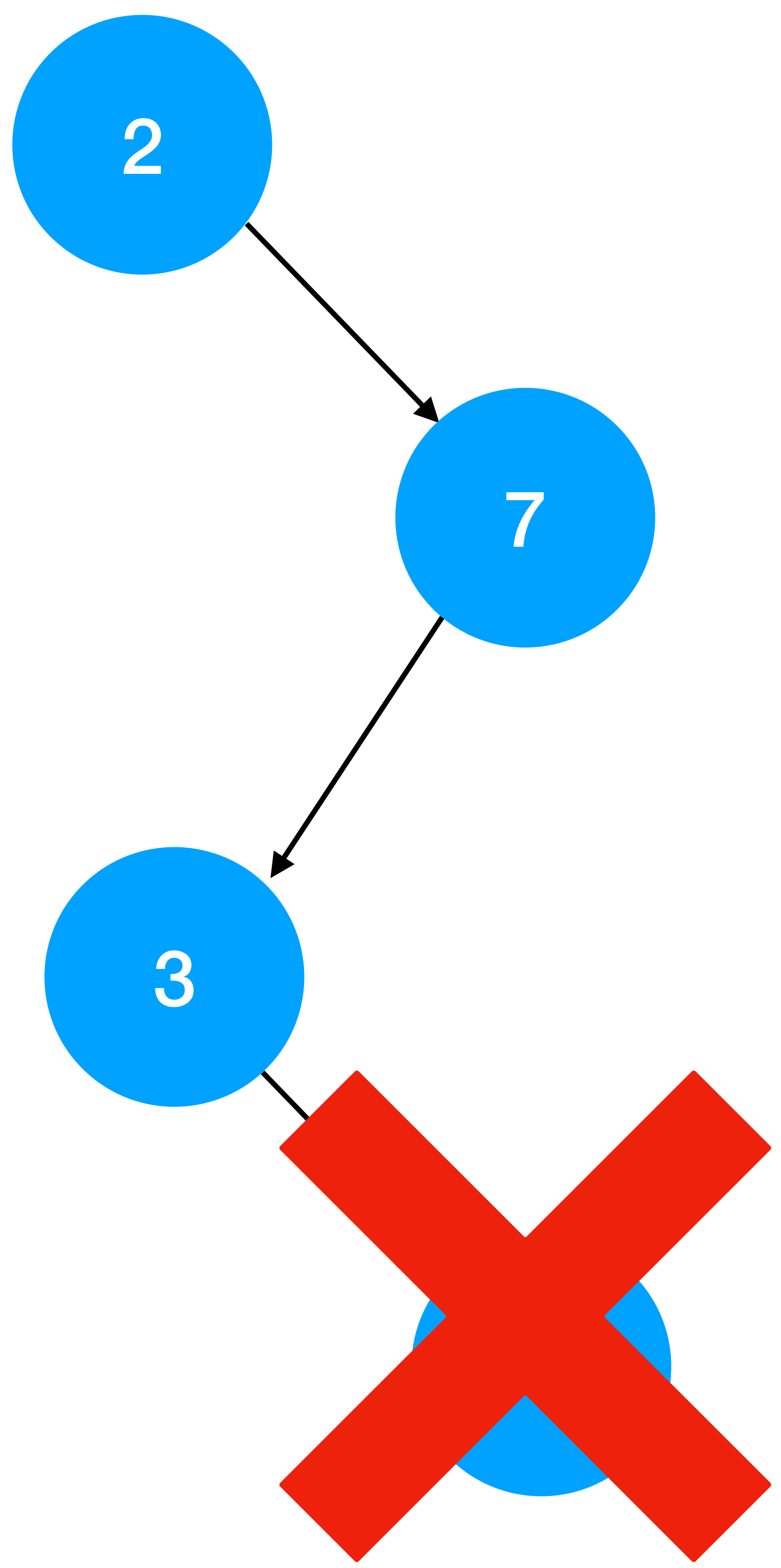
$LB = 2$

$UB = +\infty$

$3 < 7 \rightarrow UB = 7$

Question 3.1.6 VISITE POSSIBLES EN DFS

2,7,3,8,4,5



LB = 2
UB = 7

$8 > 7 \rightarrow 8 > UB$

Question 3.1.6 VISITE POSSIBLES EN DFS

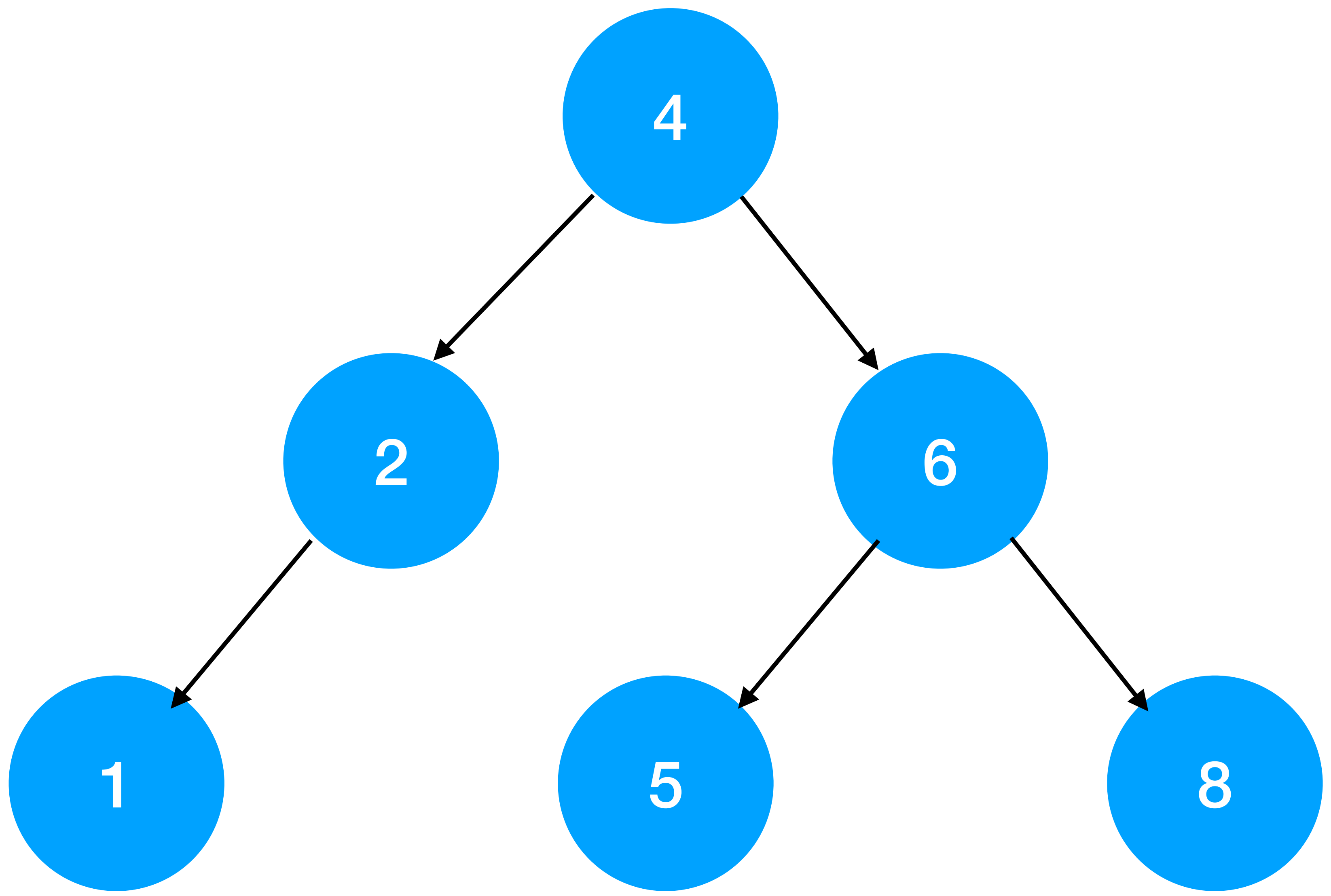
- 10,9,8,7,6,5 ✓
- 4,10,8,6,5 ✓
- 1,10,2,9,3,8,4,7,6,5 ✓
- 2,7,3,8,4,5 ✗
- 1,2,10,4,8,5 ✓

Question 3.1.8 Énumerer en ordre croissant les clés

Parcours infixe

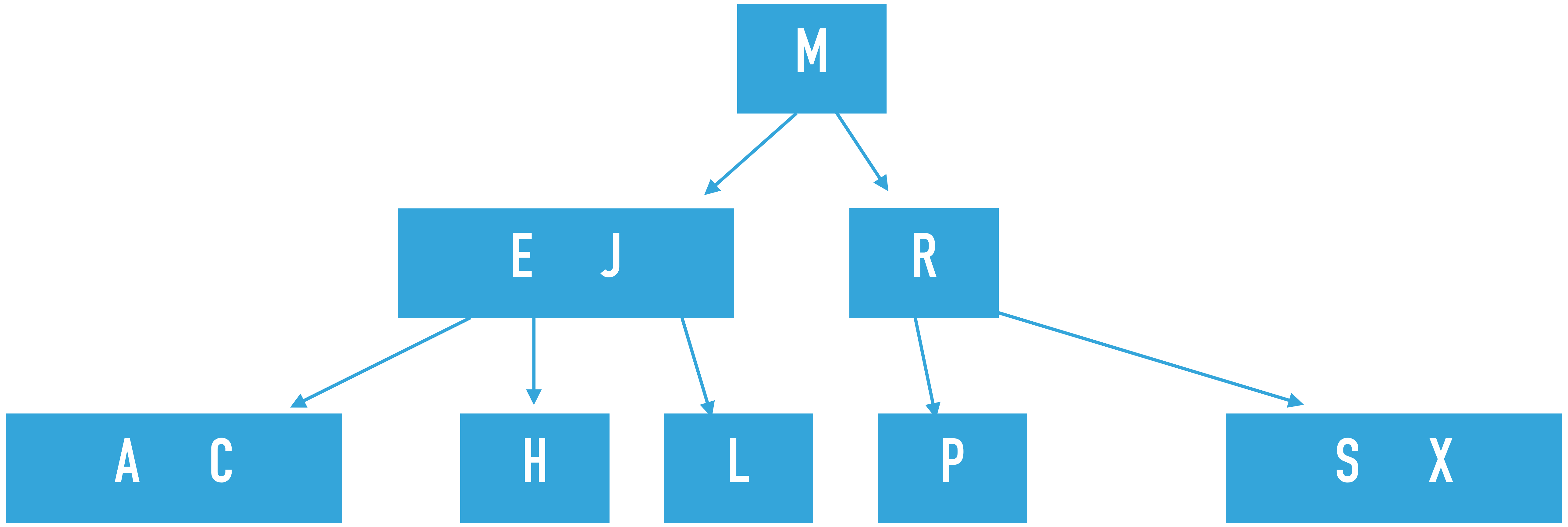
Complexité : O(nombre de noeuds)

```
public static void enumerate(Node parent) {
    enumerate(parent.leftChild)
    System.out.println(parent.key)
    enumerate(parent.rightChild)
}
```



Arbres 2-3

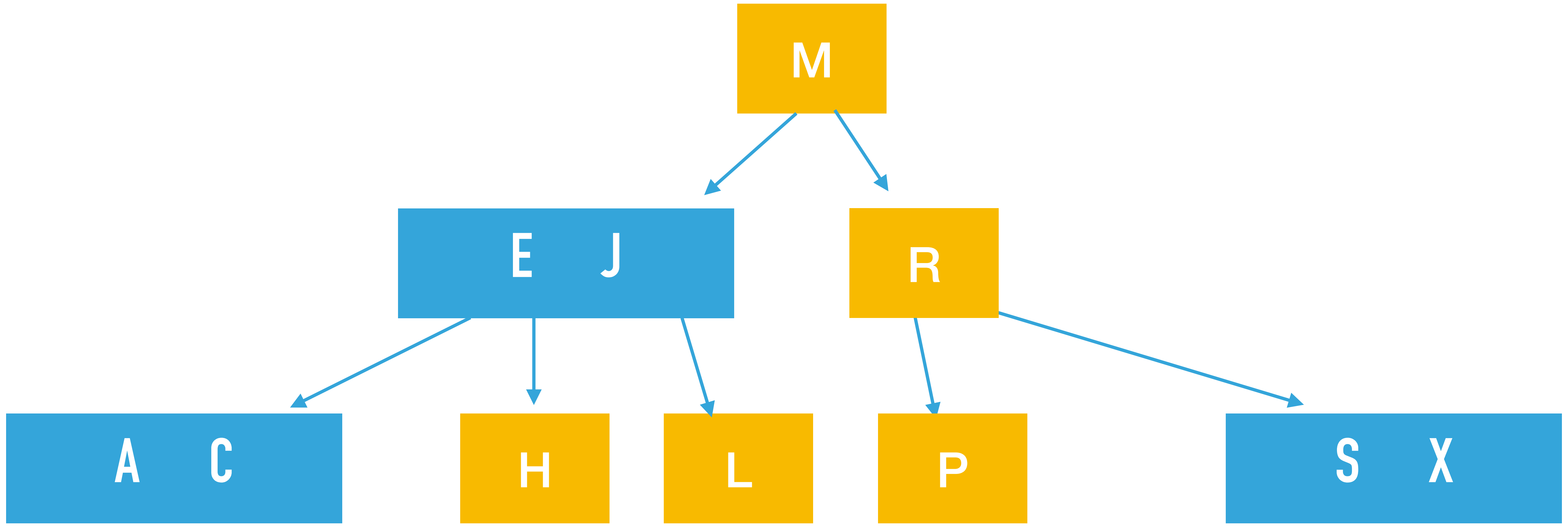
Arbre 2-3 contient soit :



Arbres 2-3

Arbre 2-3 contient soit :

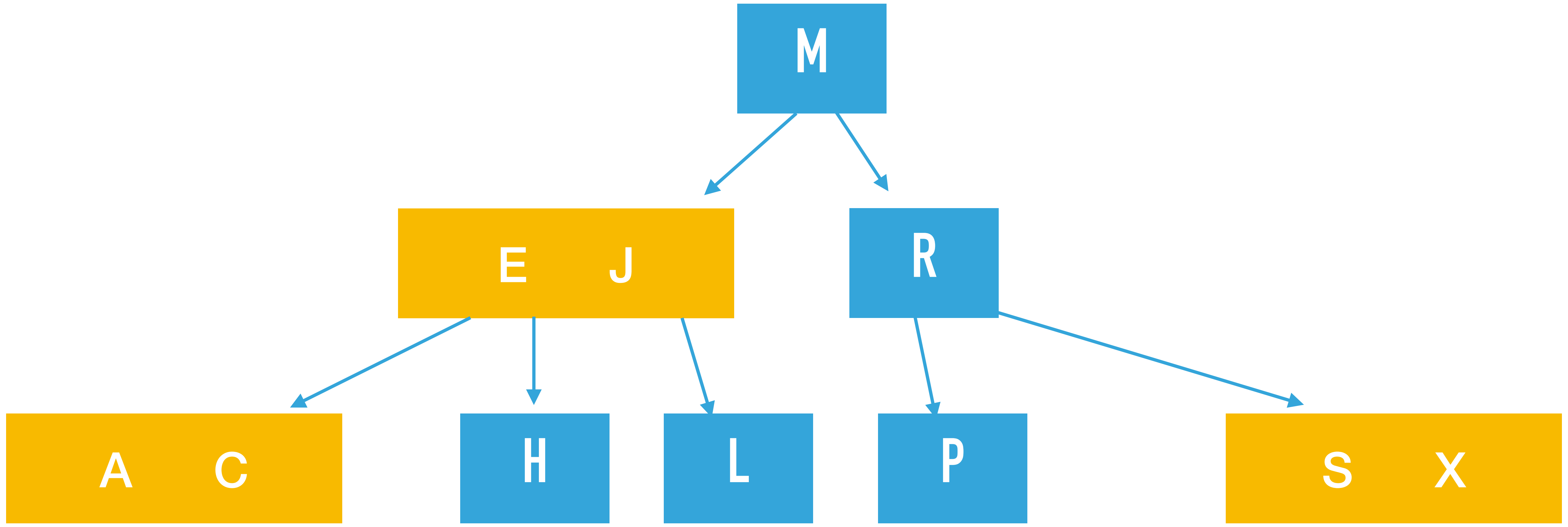
- 2-noeud, avec 1 clé et deux liens
  - À gauche : clés plus petites
  - À droite : clés plus grandes



Arbres 2-3

Arbre 2-3 contient soit :

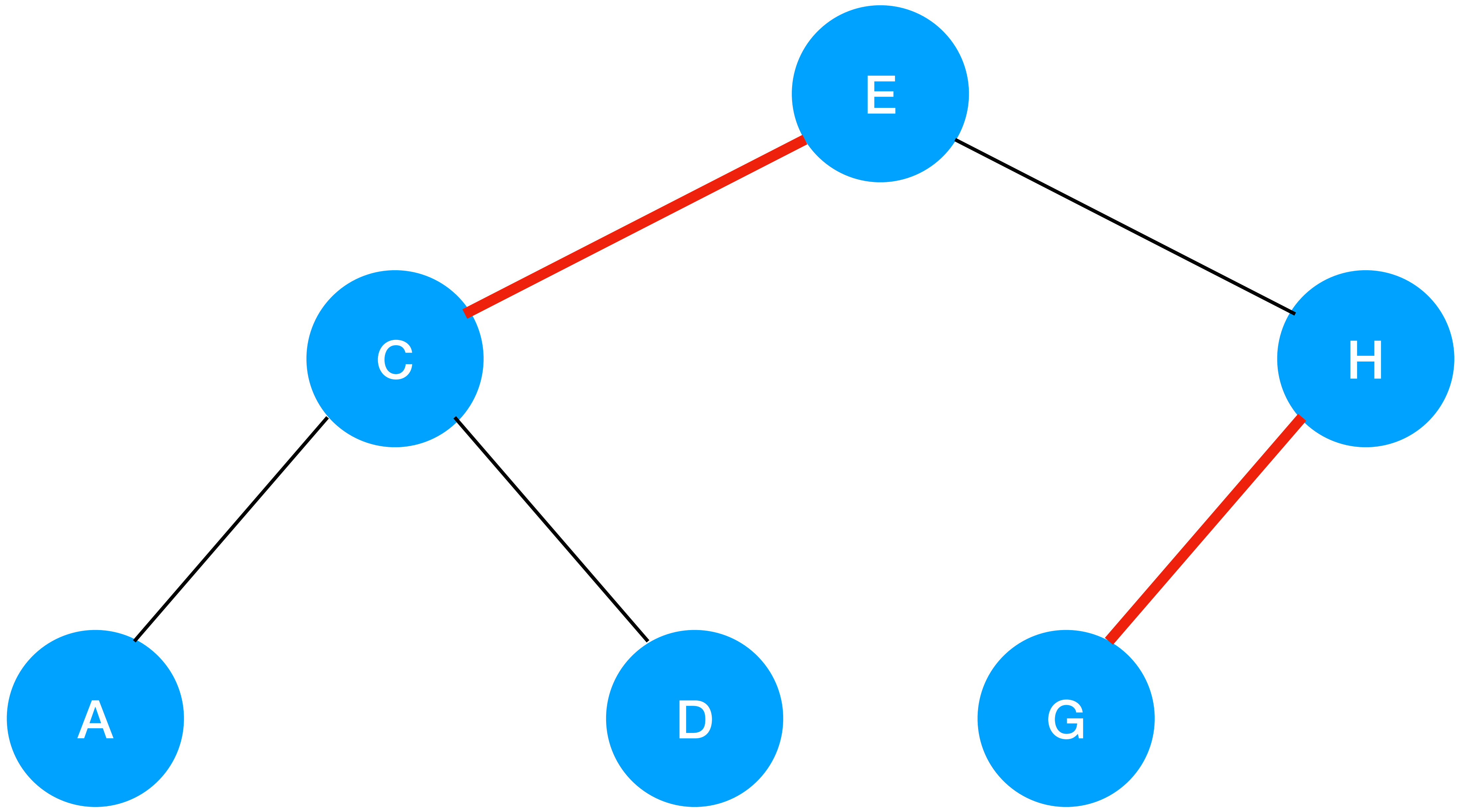
- 2-noeud, avec 1 clé et deux liens
  - À gauche : clés plus petites
  - À droite : clés plus grandes
- 3-noeud, avec 2 clés et 3 liens
  - À gauche : clés plus petites
  - À droite : clés plus grandes
  - Au milieu : clés entre



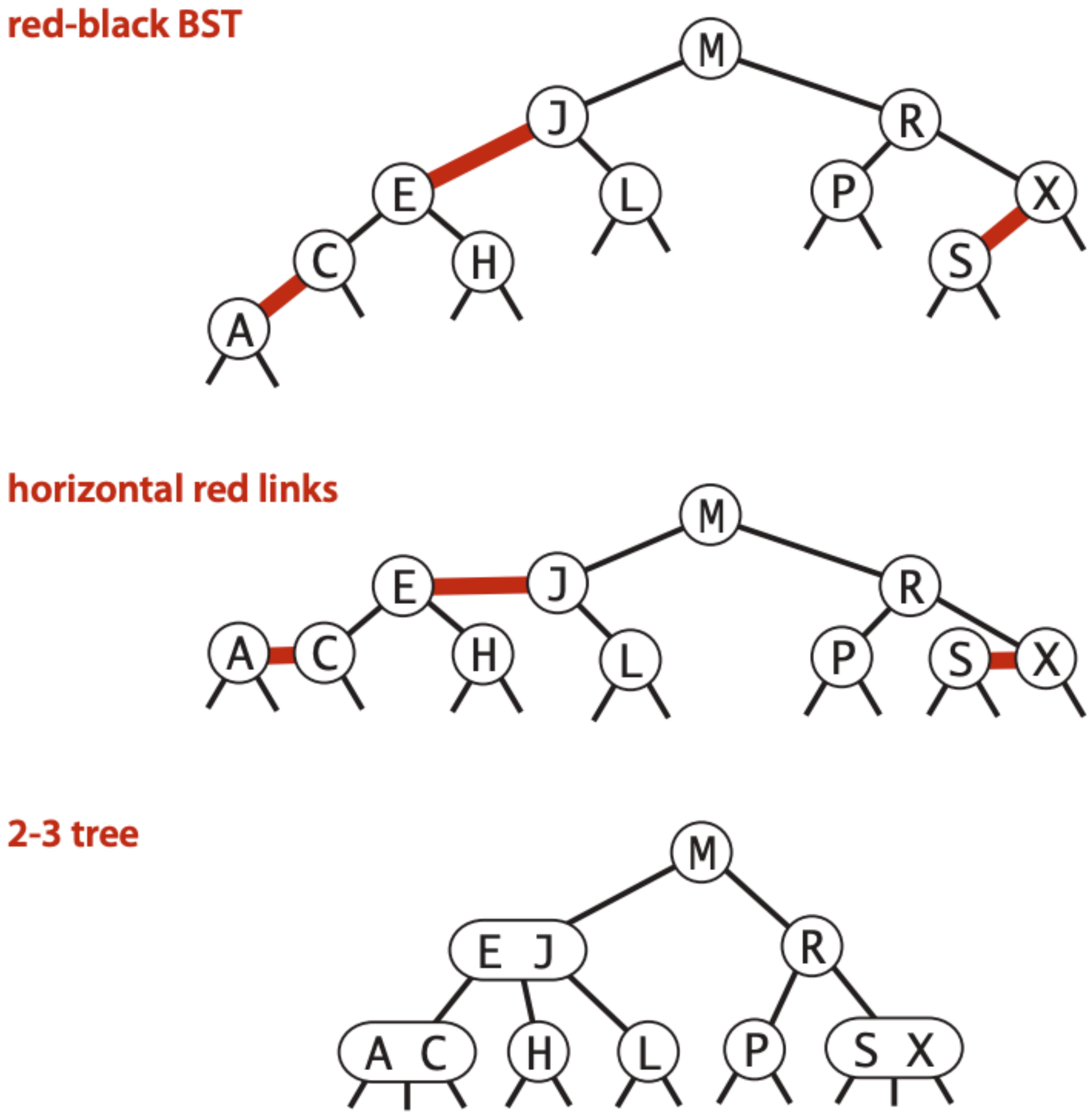
Arbre Red-Black

Arbre Red-Black :

- 2 types de liens (**rouge** ou **noir**)
- Liens **rouges** tjs à **gauche**
- **Pas** deux liens **rouges d'affilées**
- **Equilibre parfait** sur les liens **noirs**



Correspondance 2-3 et Red-Black



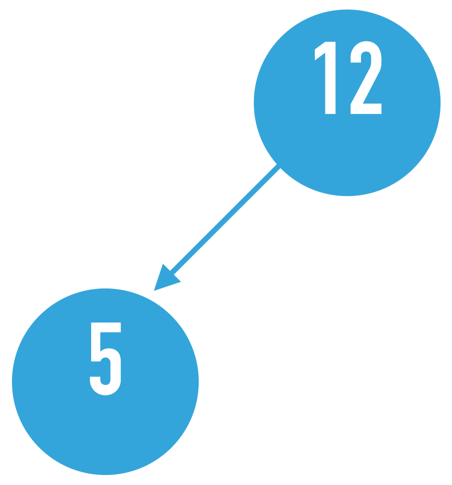
Question 3.1.9 insertions

12, 5, 10, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions

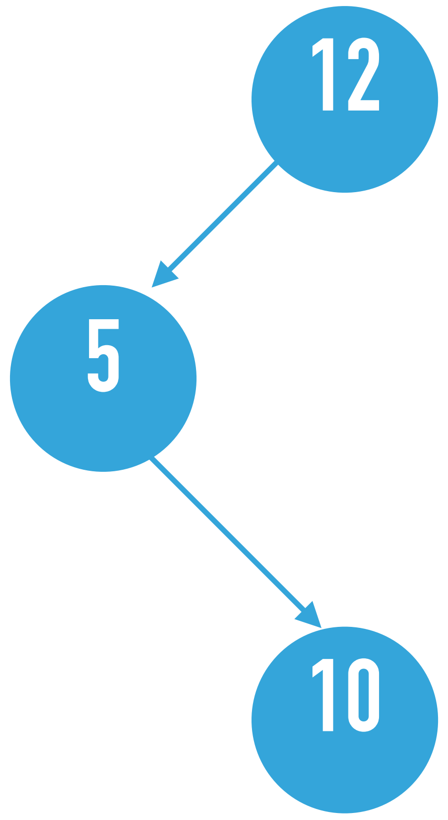
~~12~~, 5, 10, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



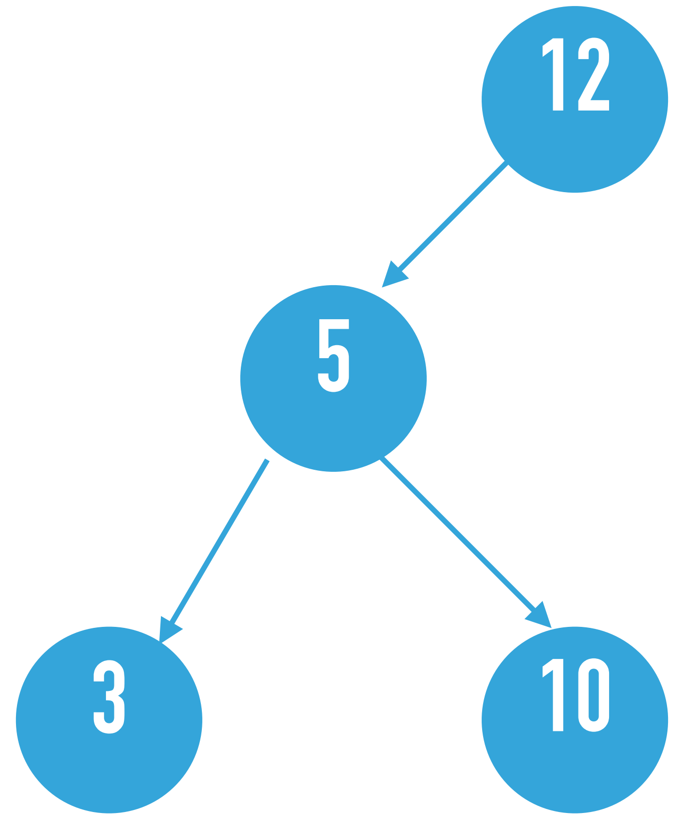
~~12, 5~~, 10, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



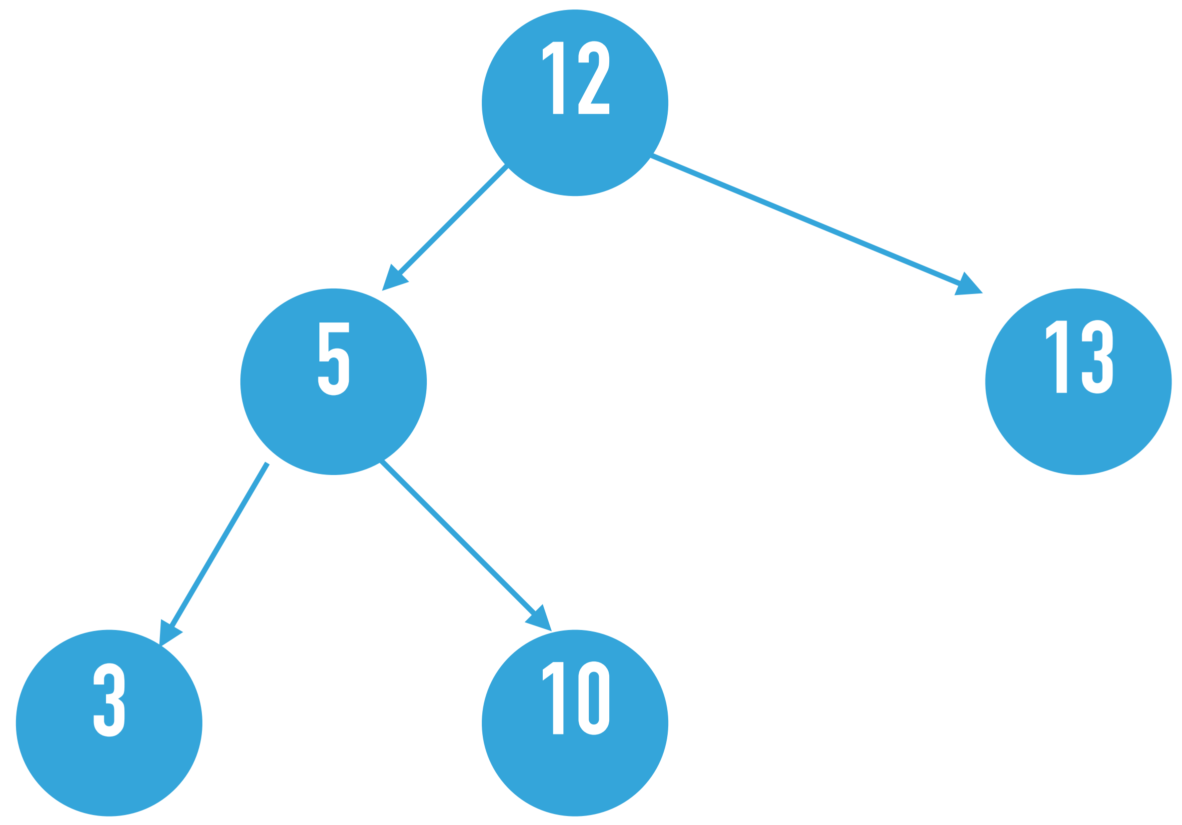
~~12, 5, 10~~, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



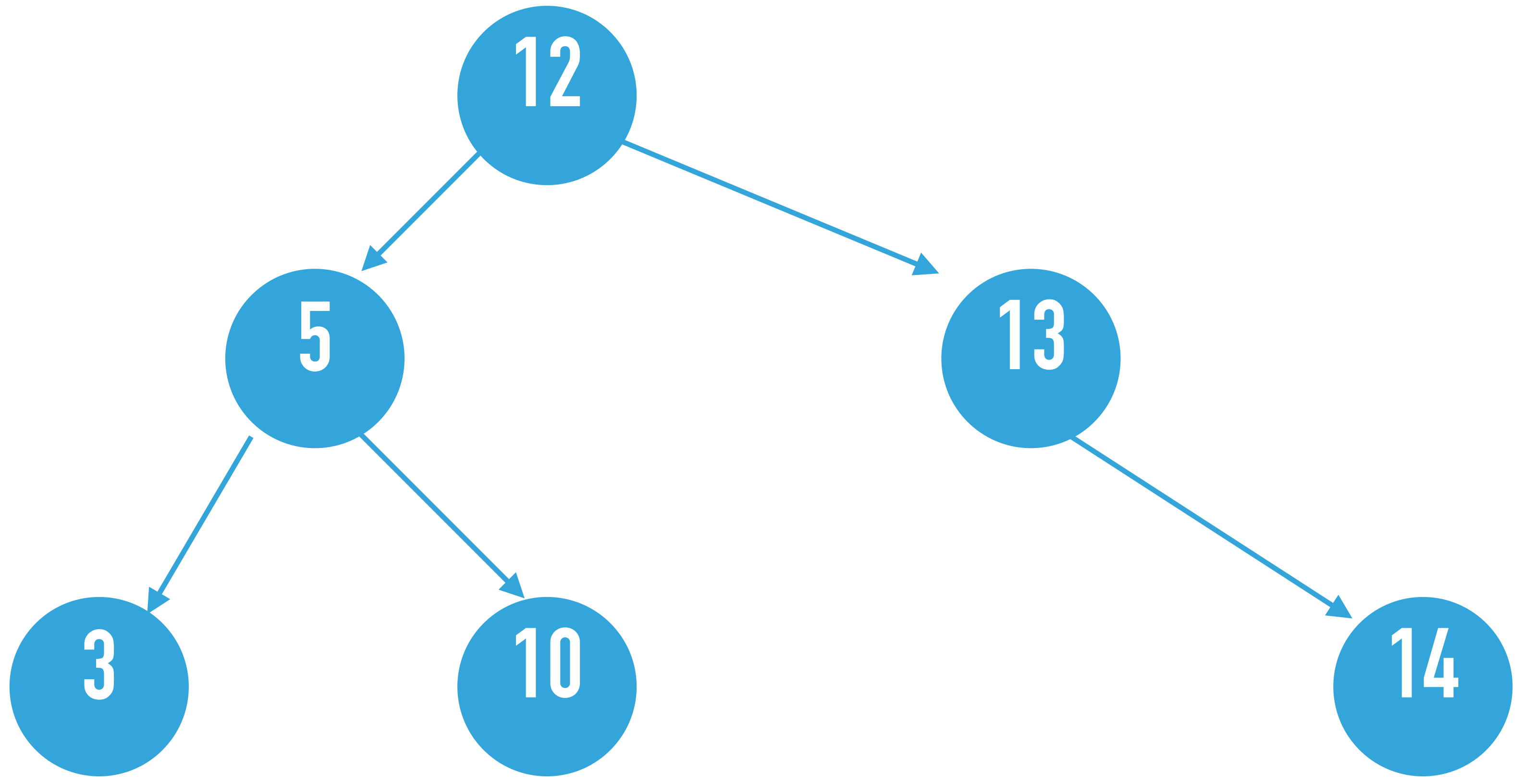
~~12, 5, 10, 3~~, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



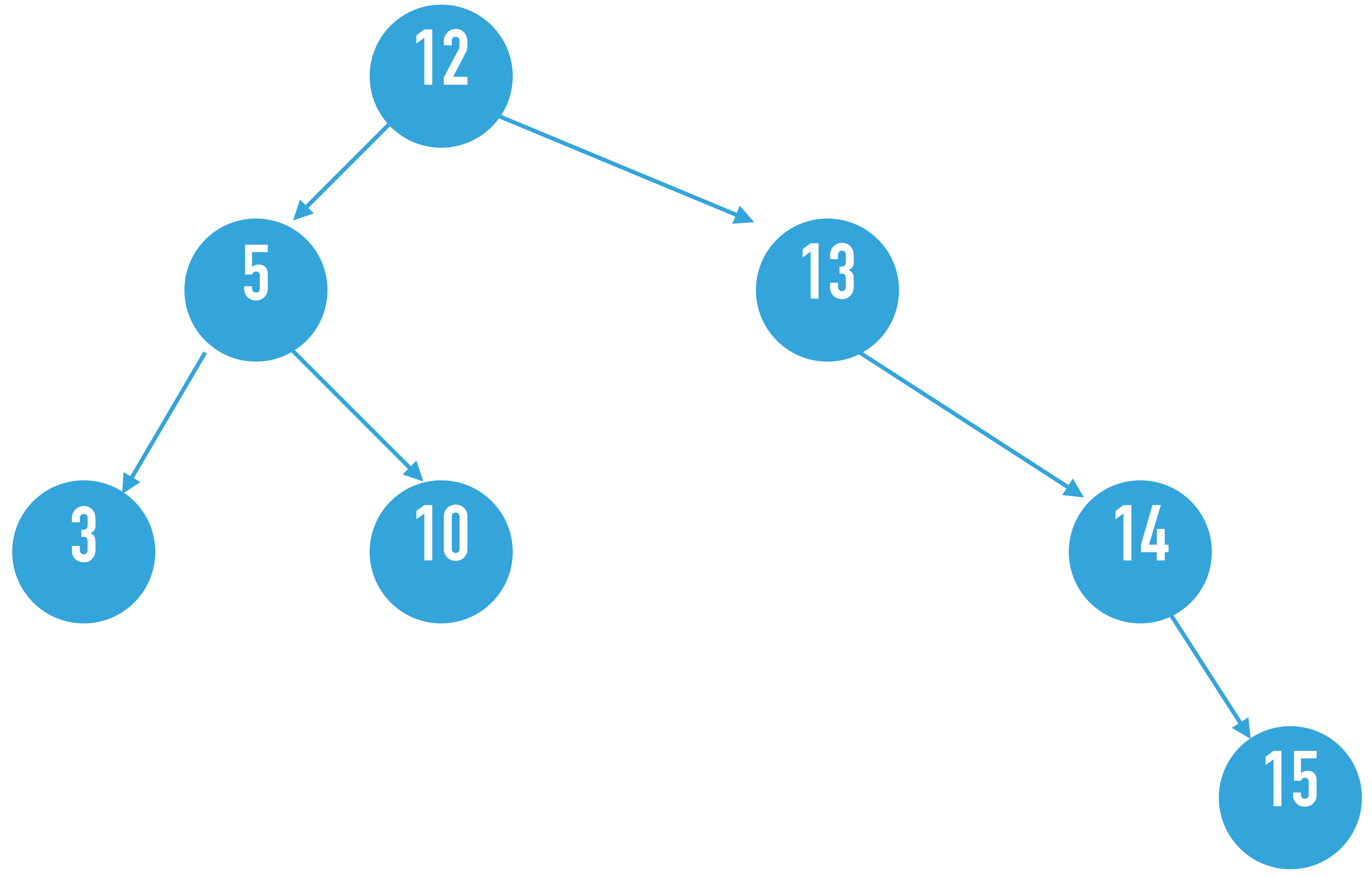
~~12, 5, 10, 3, 13~~, 14, 15, 17, 18, 15

Question 3.1.9 insertions



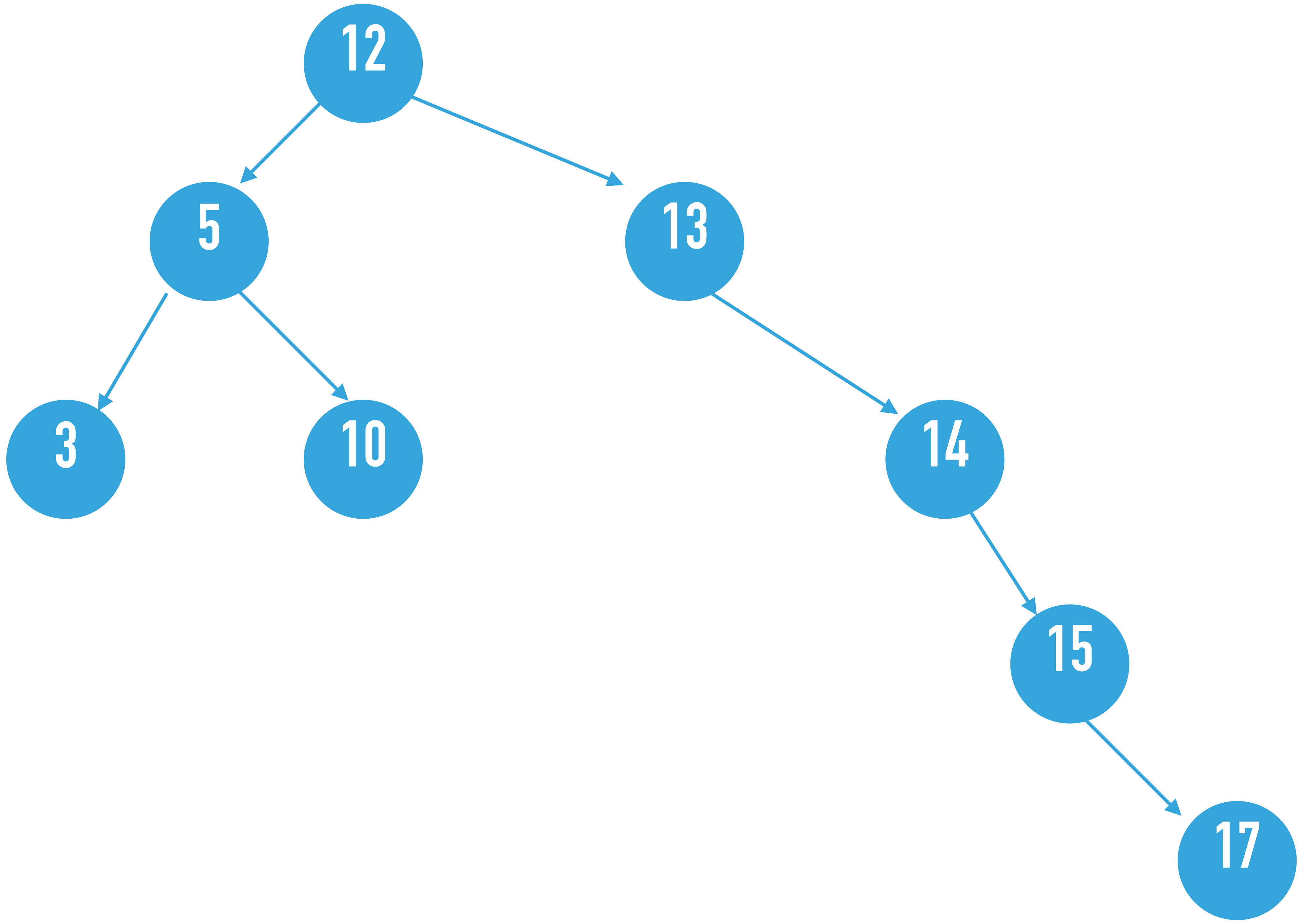
~~12, 5, 10, 3, 13, 14~~, 15, 17, 18, 15

Question 3.1.9 insertions



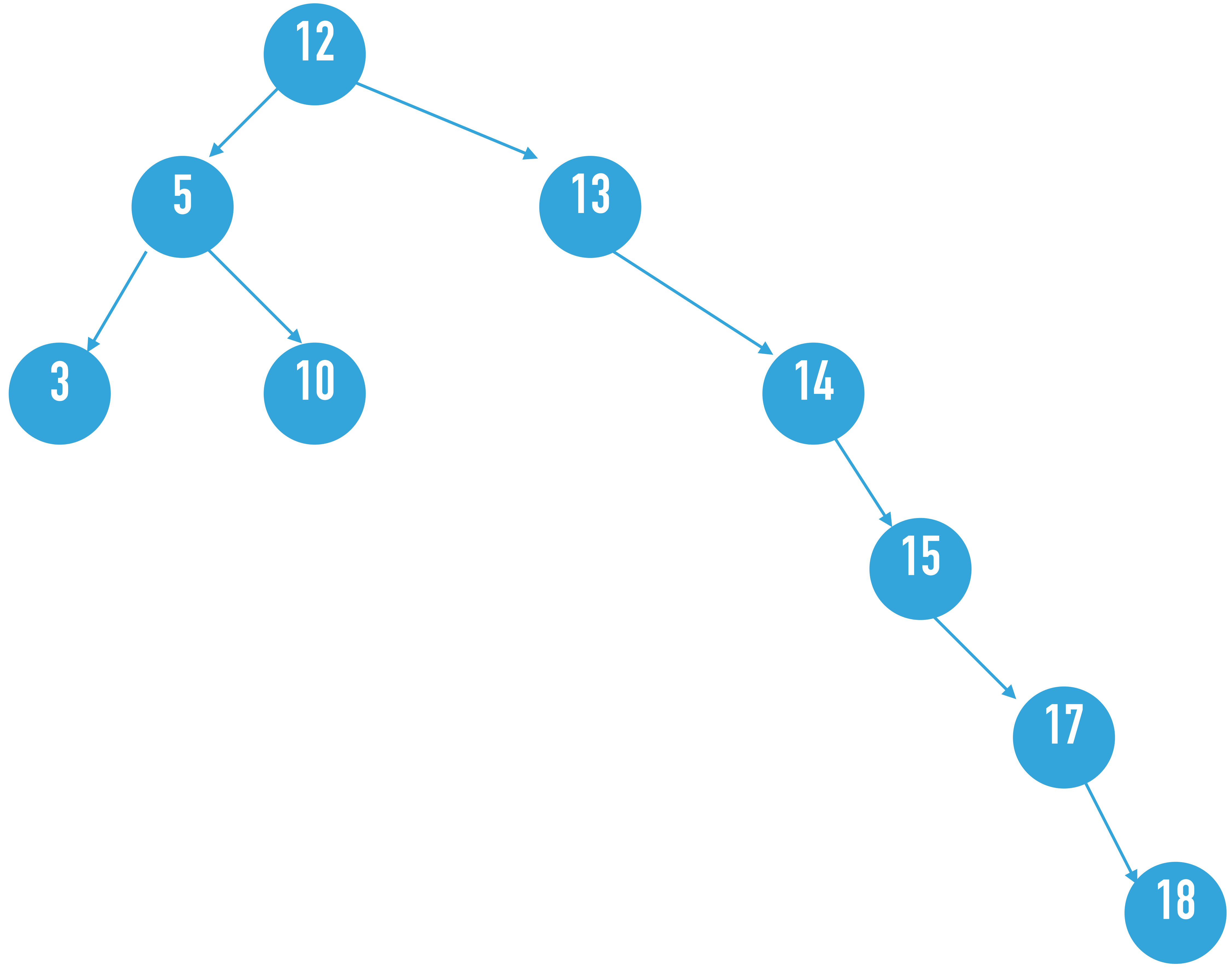
~~12, 5, 10, 3, 13, 14, 15~~, 17, 18, 15

Question 3.1.9 insertions



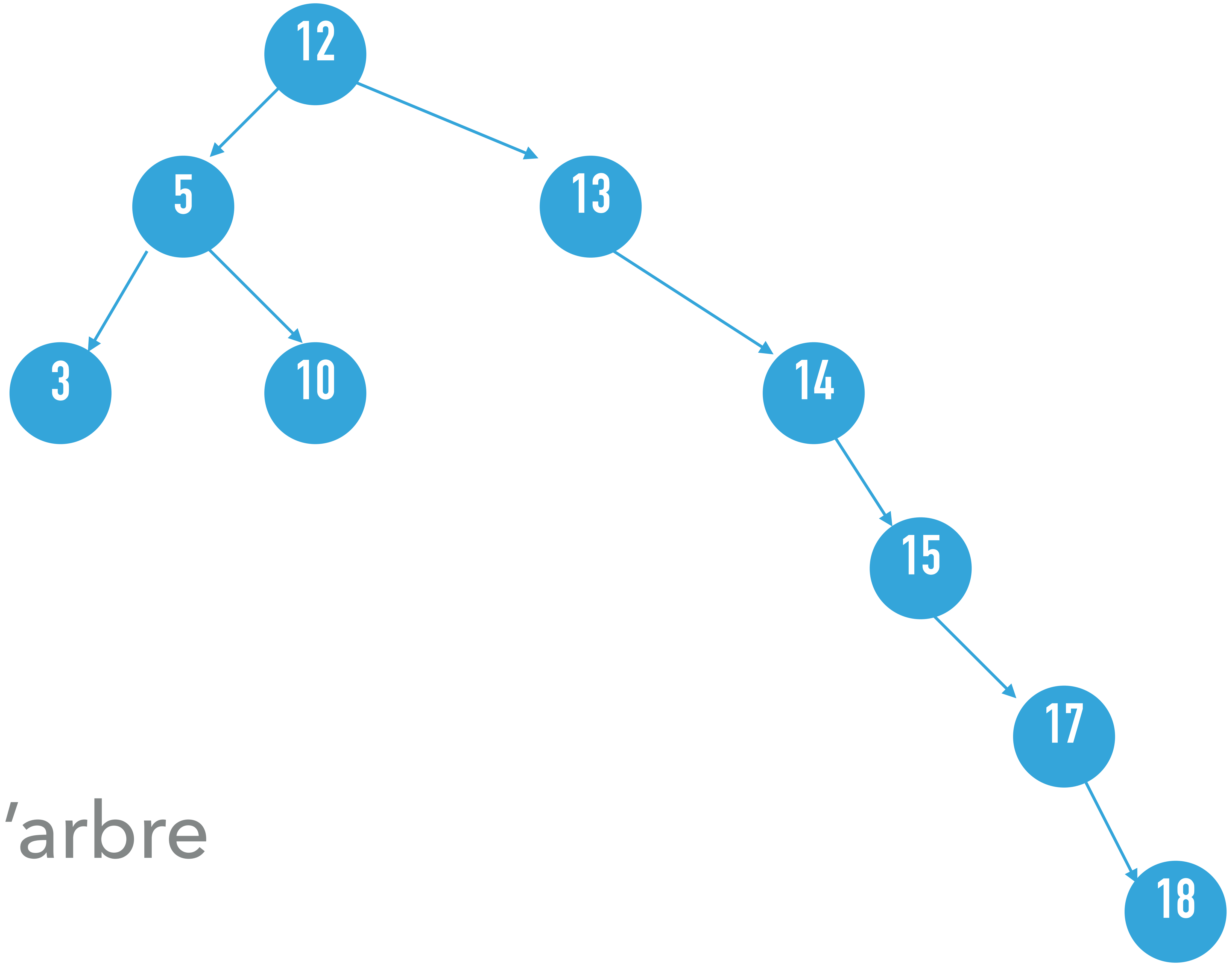
~~12, 5, 10, 3, 13, 14, 15, 17~~, 18, 15

Question 3.1.9 insertions



~~12, 5, 10, 3, 13, 14, 15, 17, 18~~, 15

Question 3.1.9 insertions



15 est déjà dans l'arbre

~~12, 5, 10, 3, 13, 14, 15, 17, 18, 15~~

Question 3.1.9 insertions

12

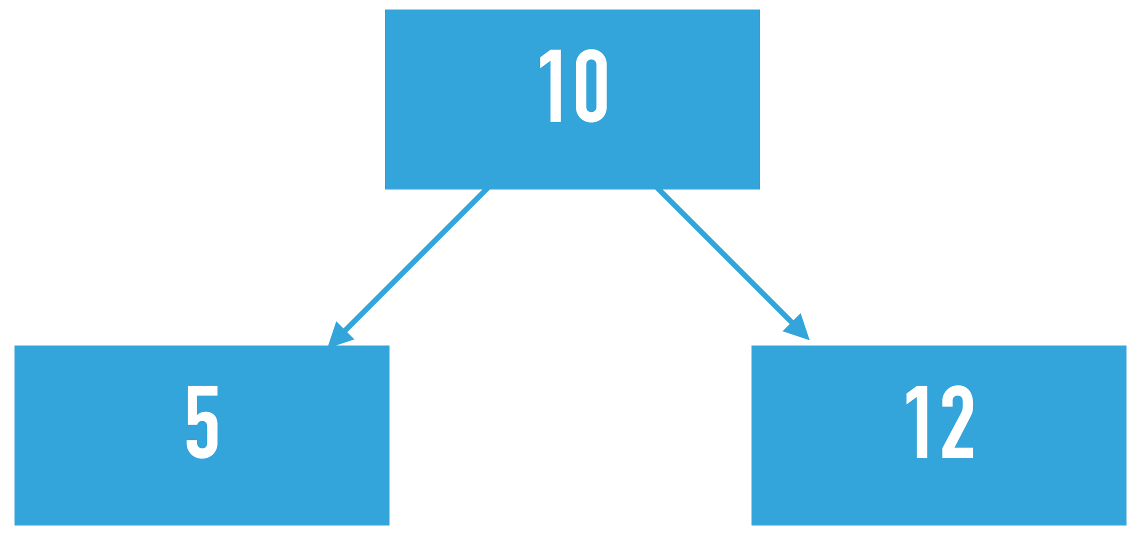
~~12~~, 5, 10, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions

5	12

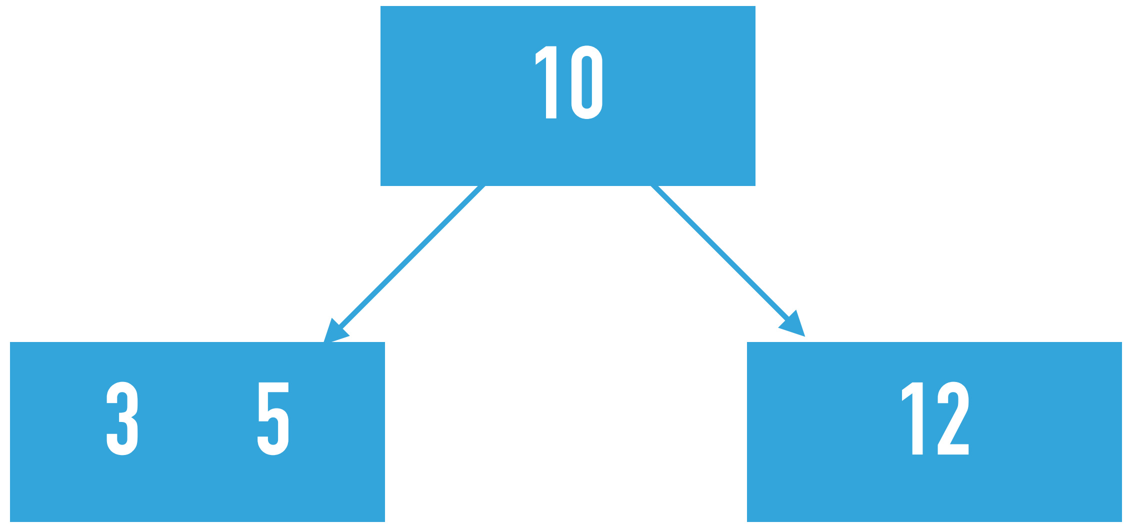
~~12~~, ~~5~~, 10, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



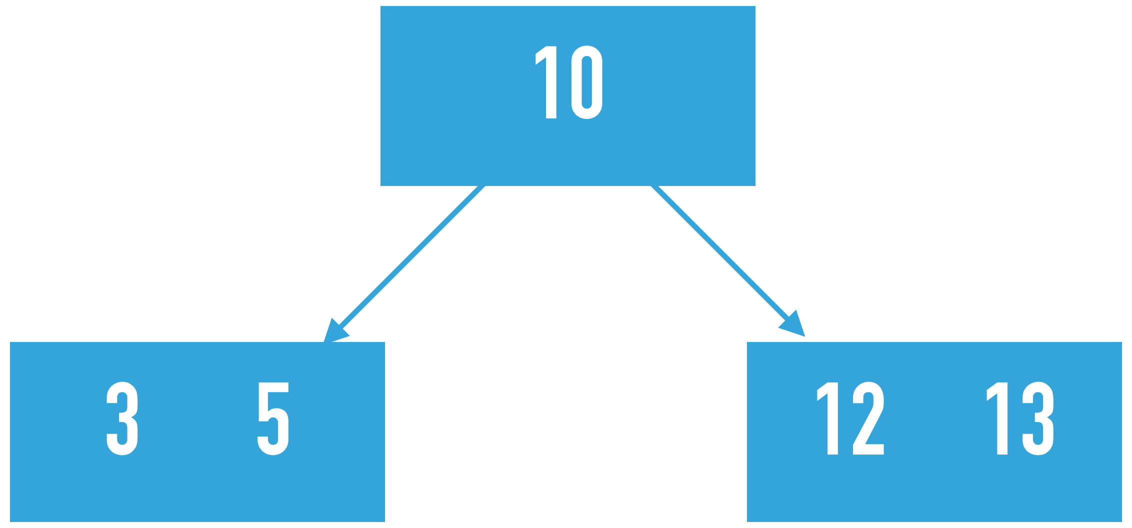
~~12, 5, 10~~, 3, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



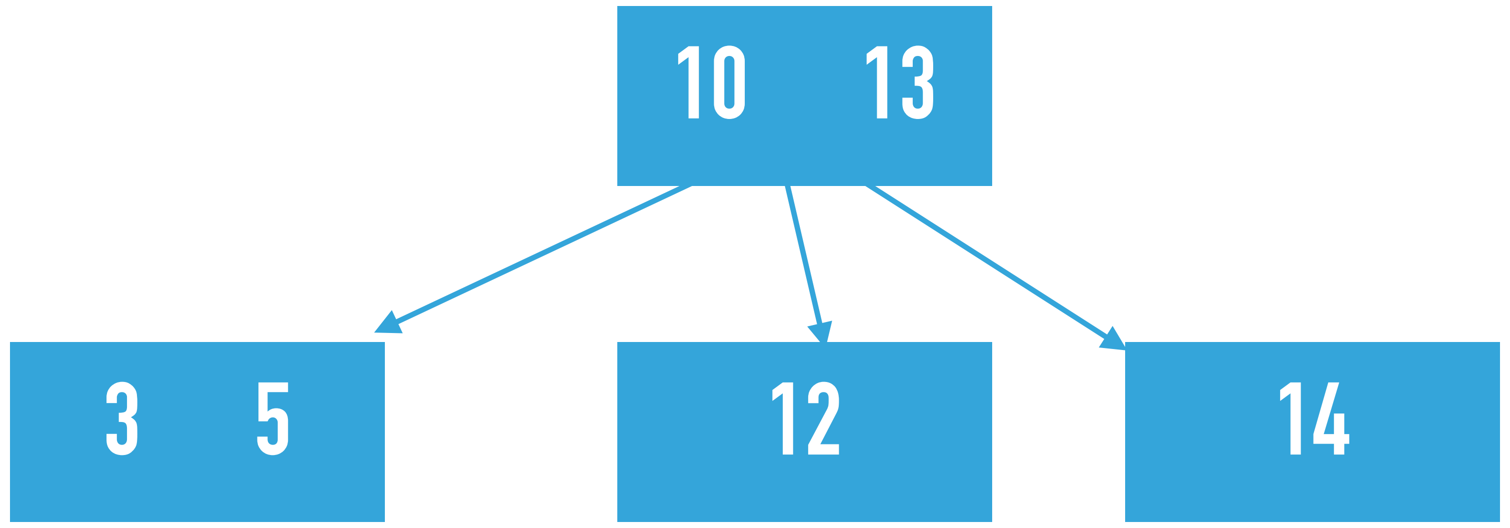
~~12, 5, 10, 3~~, 13, 14, 15, 17, 18, 15

Question 3.1.9 insertions



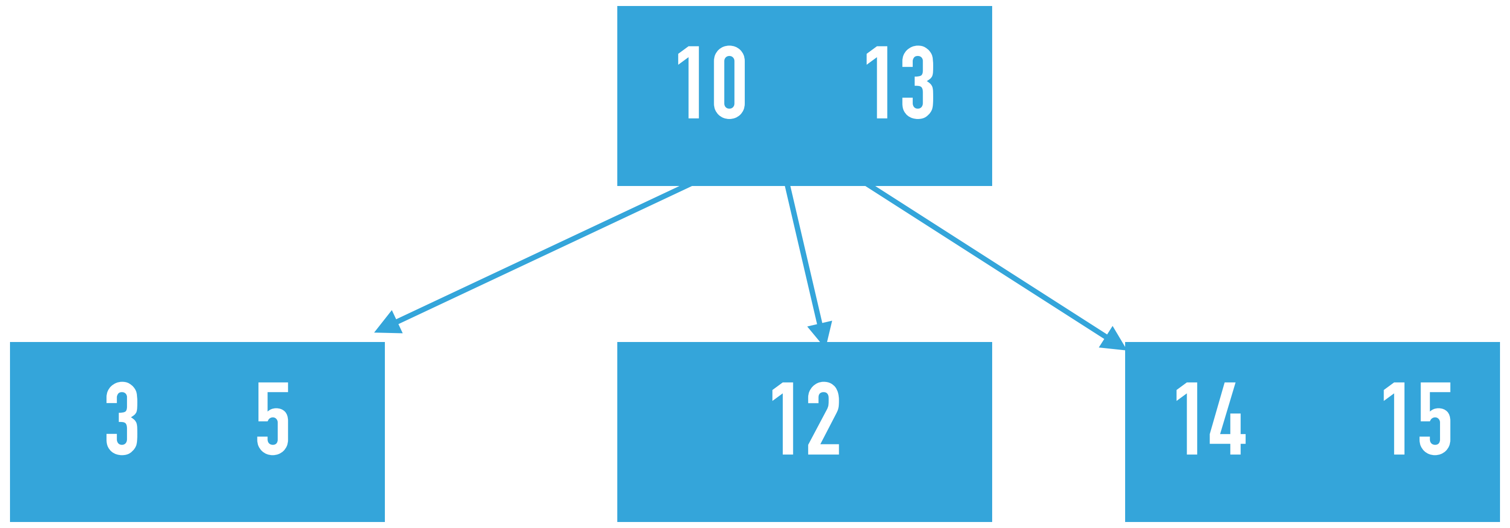
~~12, 5, 10, 3, 13~~, 14, 15, 17, 18, 15

Question 3.1.9 insertions



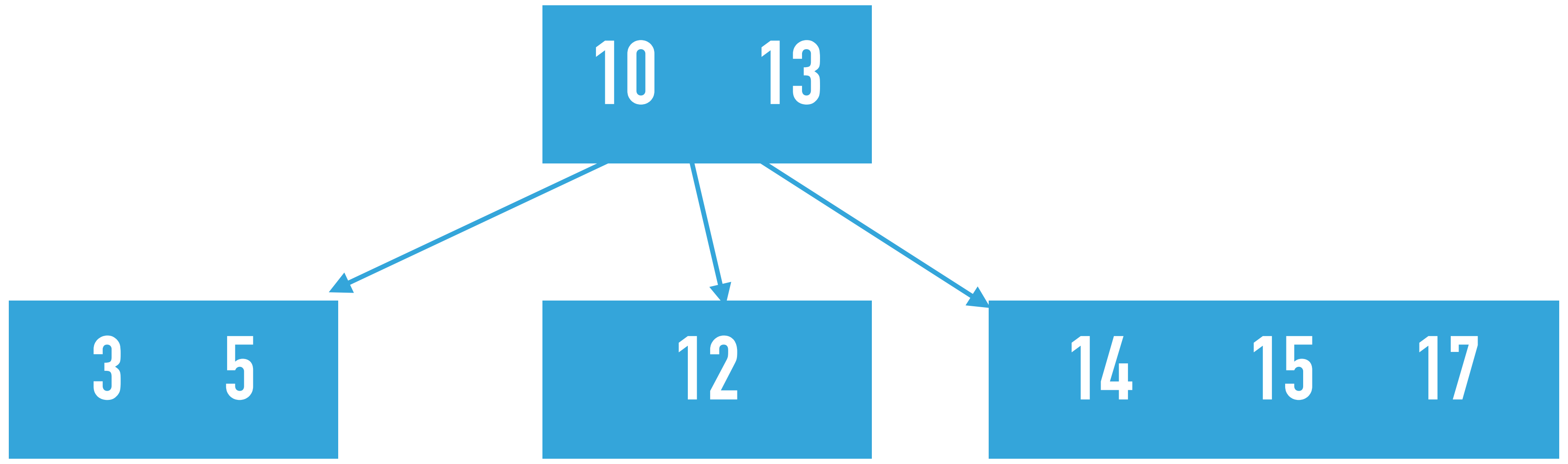
~~12, 5, 10, 3, 13, 14~~, 15, 17, 18, 15

Question 3.1.9 insertions



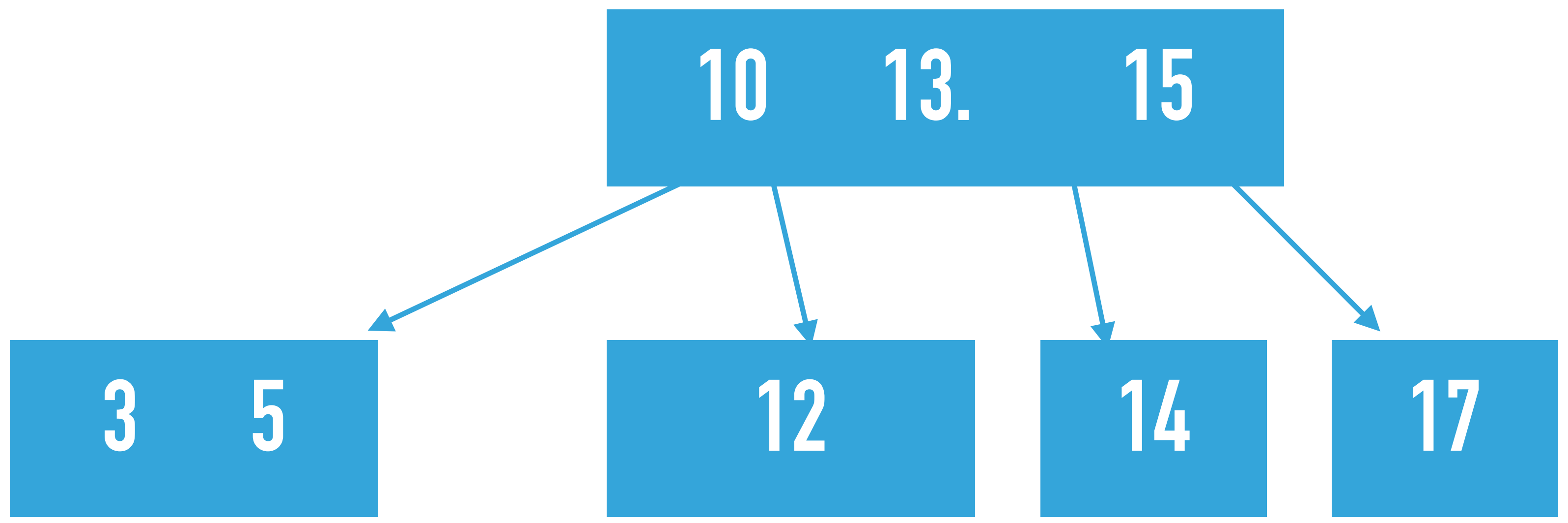
~~12, 5, 10, 3, 13, 14, 15~~, 17, 18, 15

Question 3.1.9 insertions



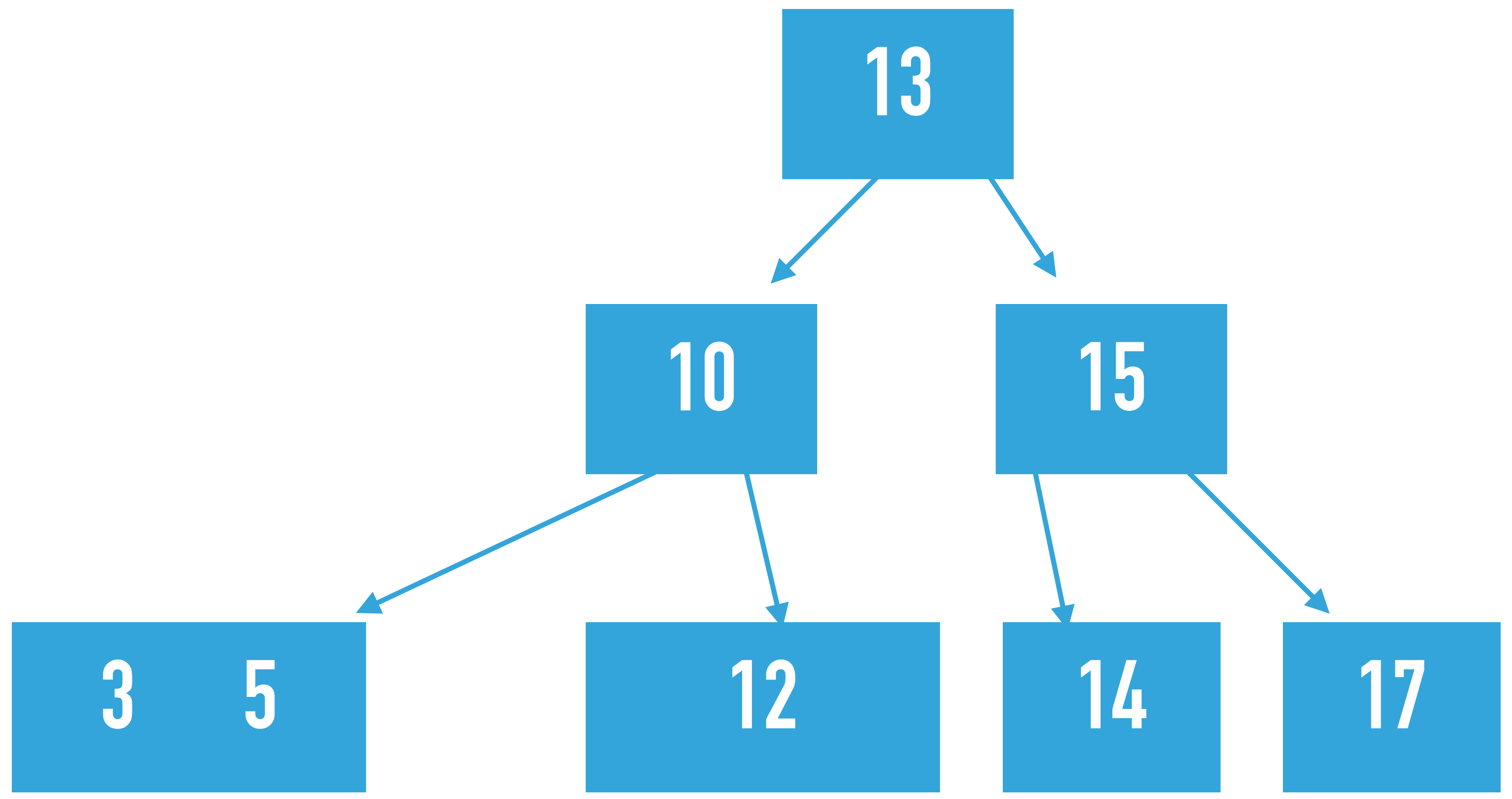
~~12, 5, 10, 3, 13, 14, 15, 17~~, 18, 15

Question 3.1.9 insertions



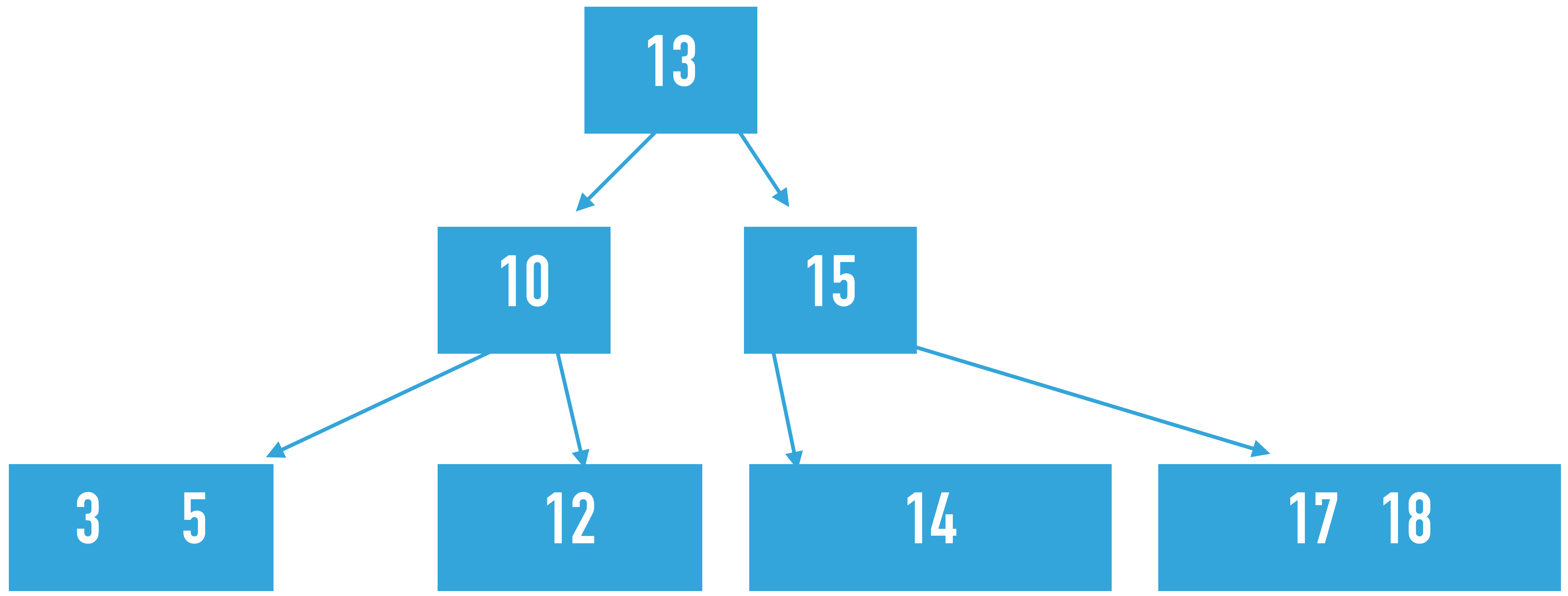
~~12, 5, 10, 3, 13, 14, 15, 17~~, 18, 15

Question 3.1.9 insertions



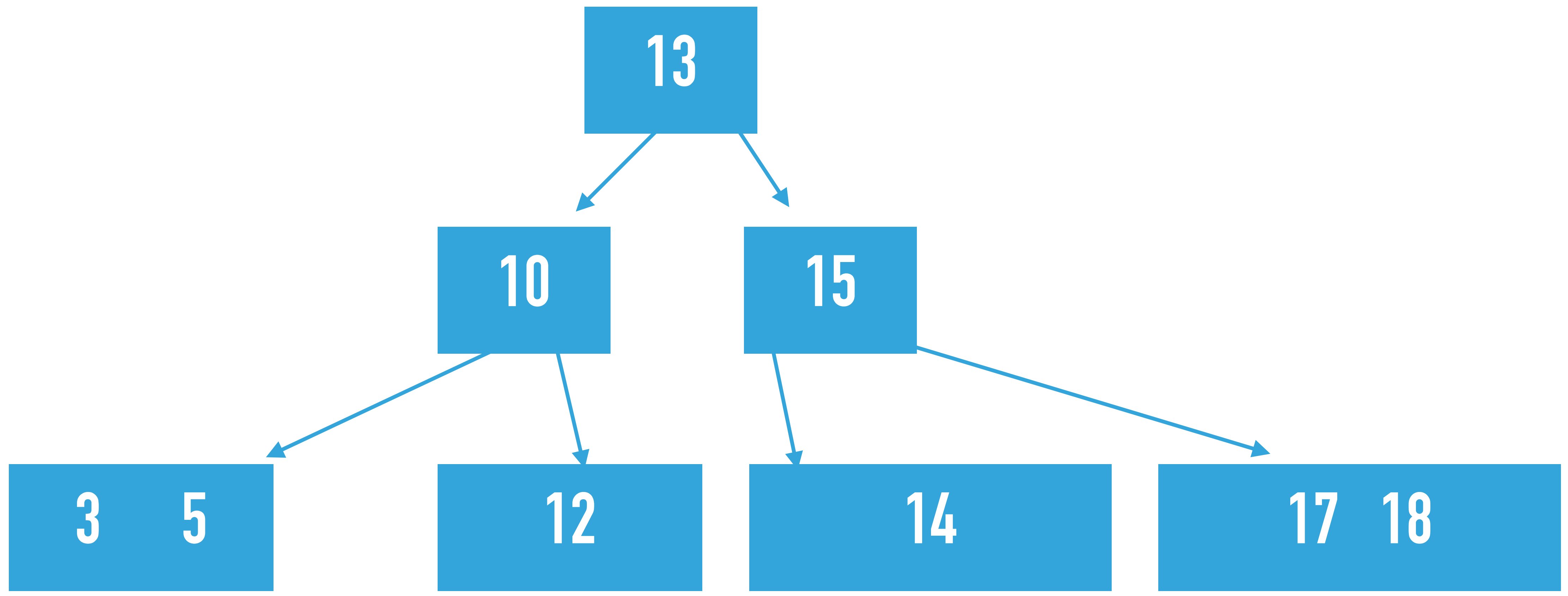
~~12, 5, 10, 3, 13, 14, 15, 17~~, 18, 15

Question 3.1.9 insertions



~~12, 5, 10, 3, 13, 14, 15, 17, 18~~, 15

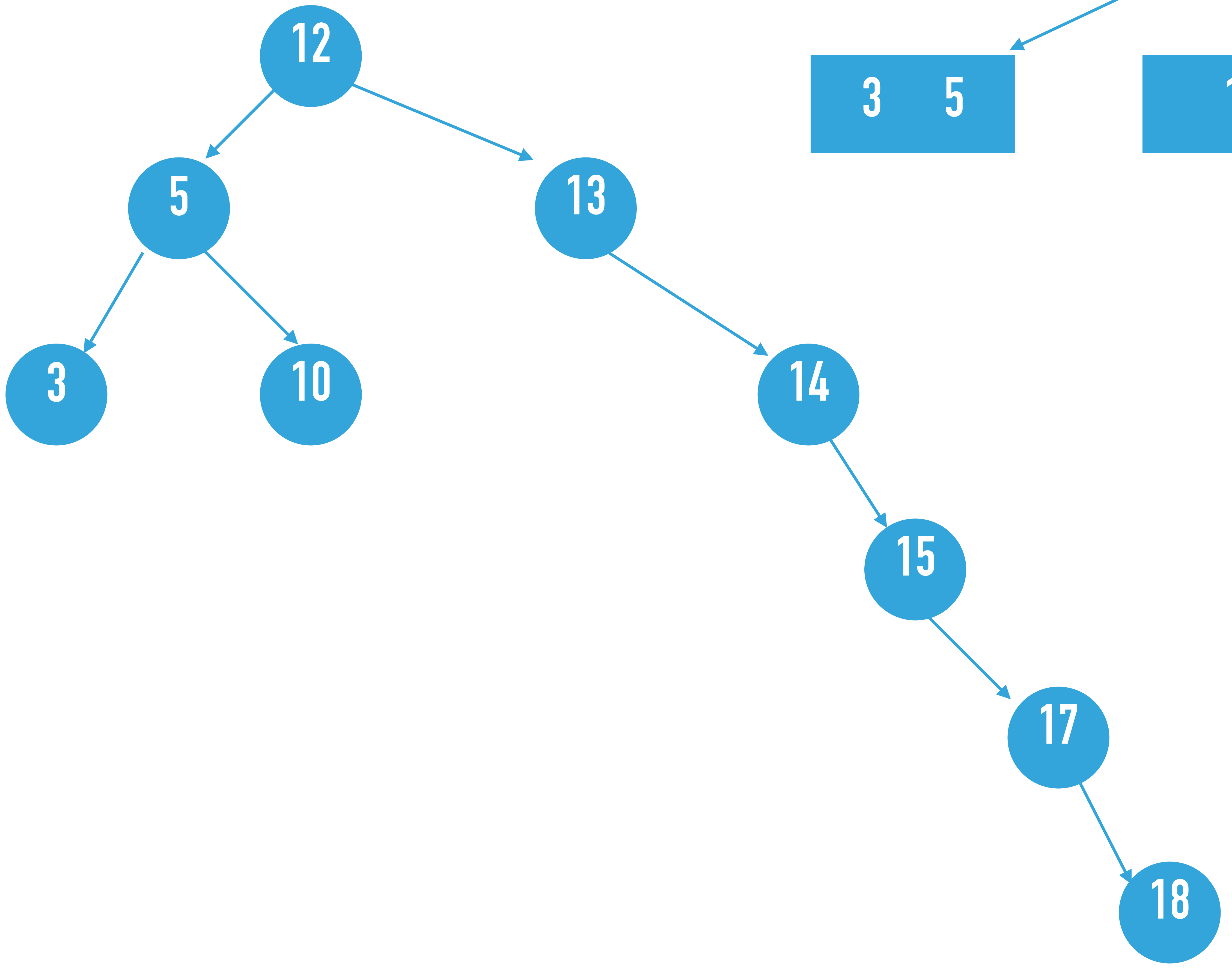
Question 3.1.9 insertions

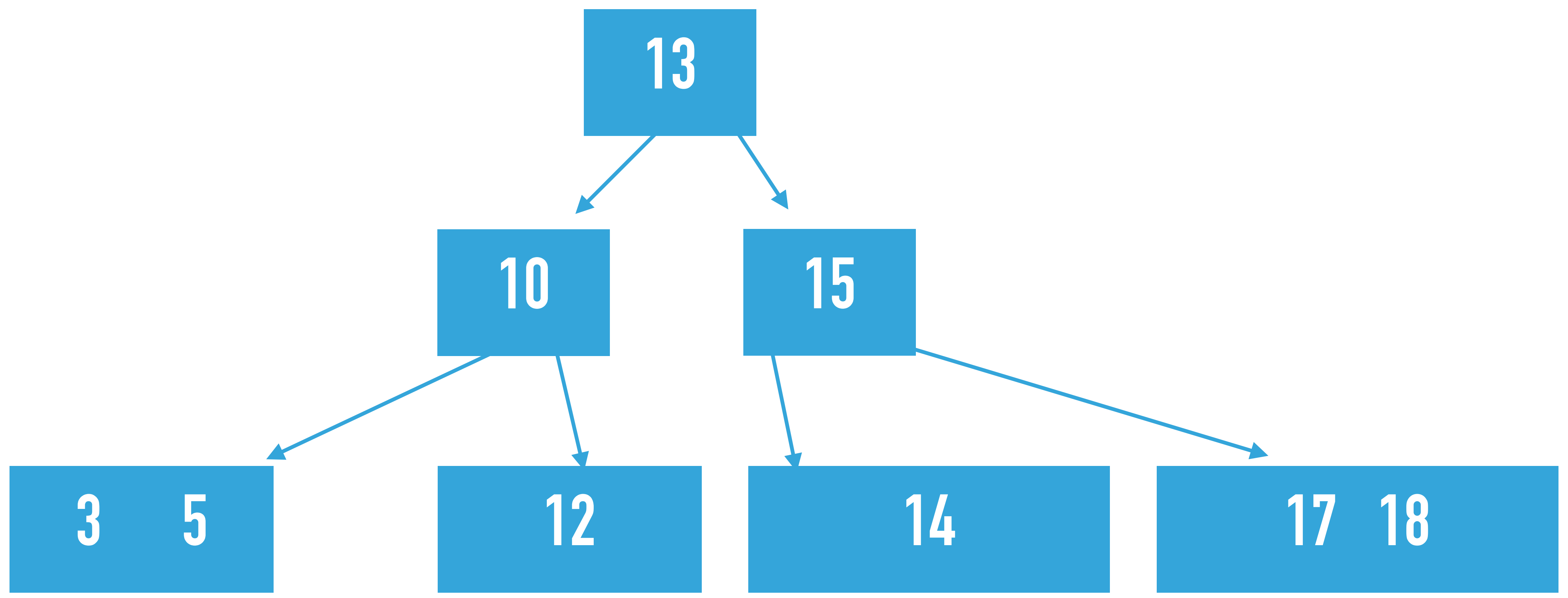


15 est déjà dans l'arbre

~~12, 5, 10, 3, 13, 14, 15, 17, 18, 15~~

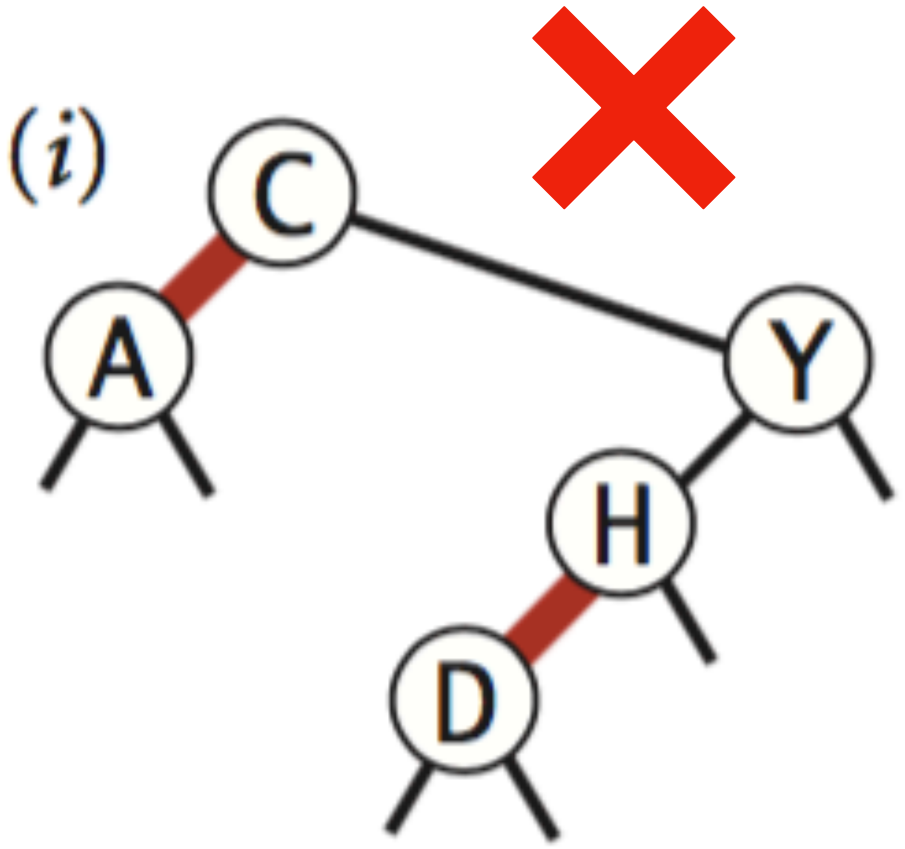
Question 3.1.9 insertions



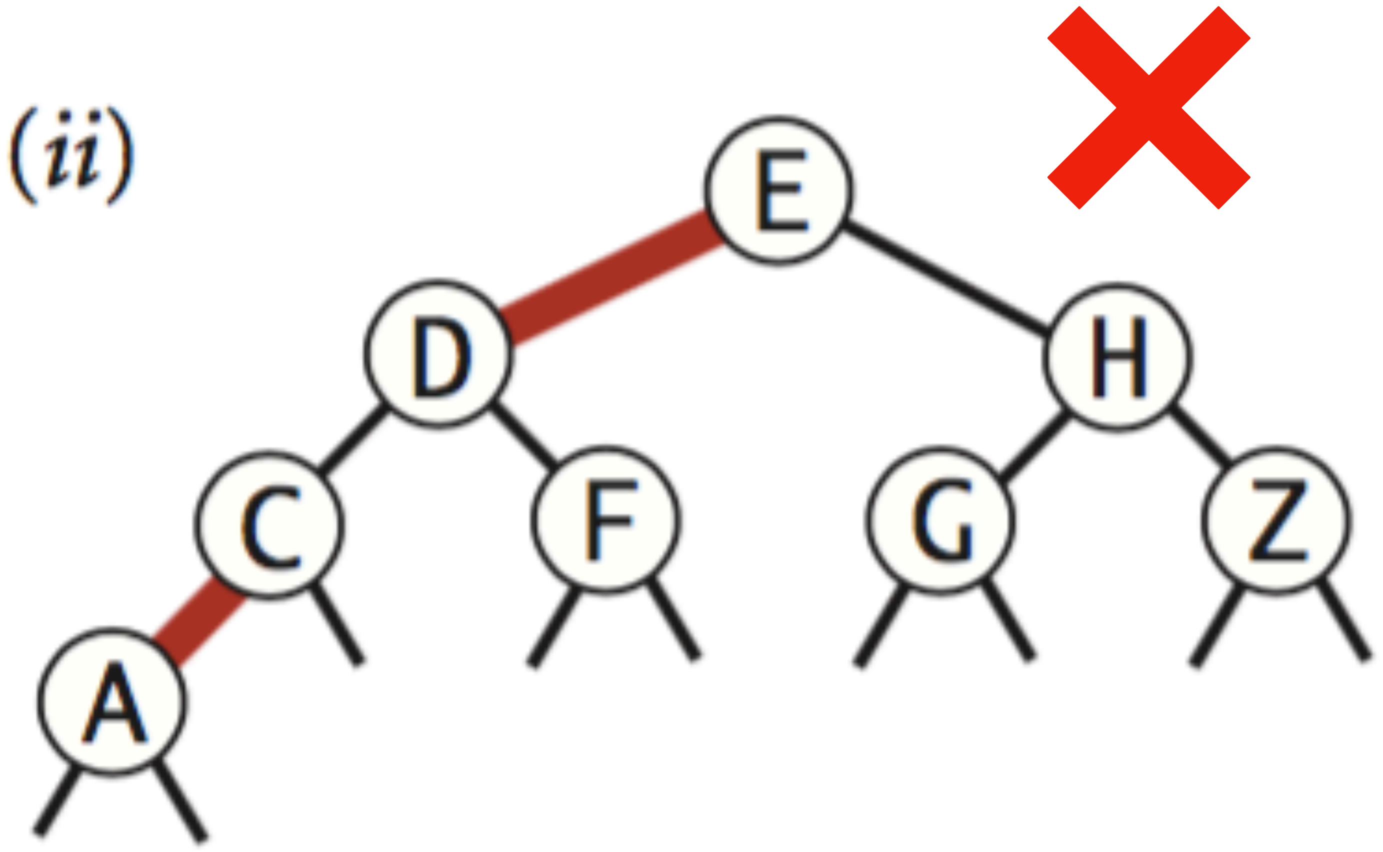


Question 3.1.10 RB?

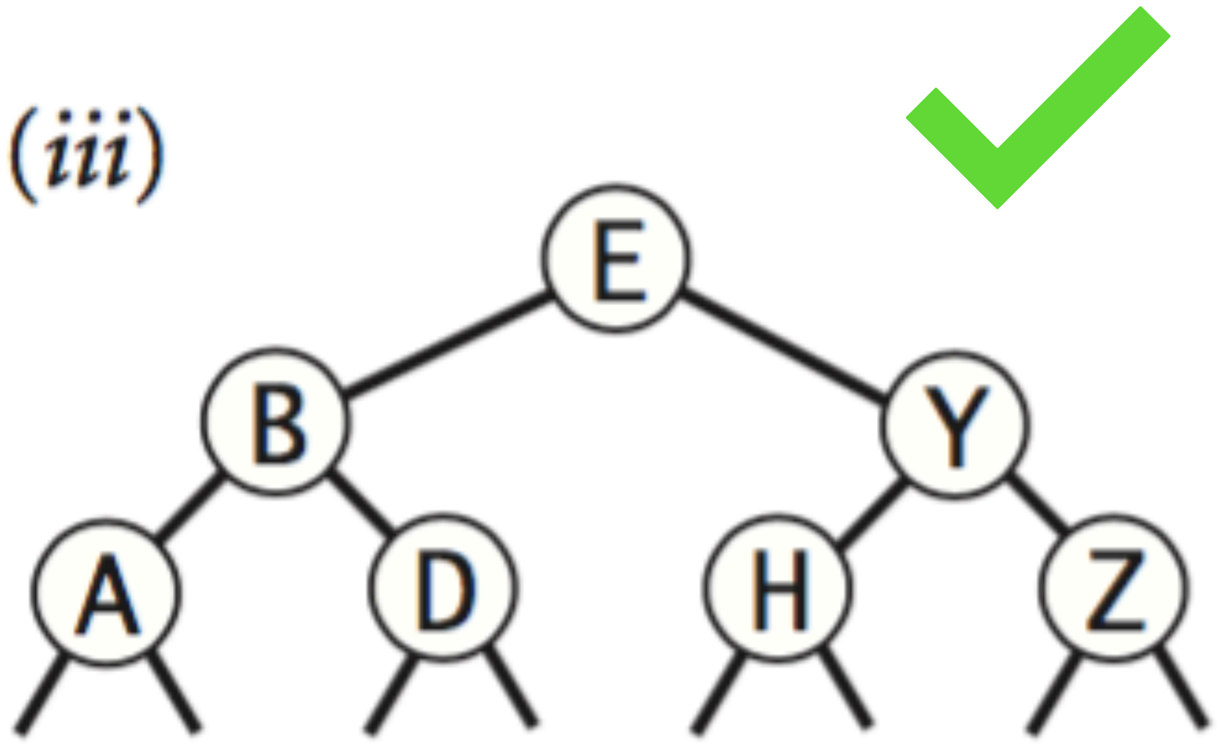
(i)



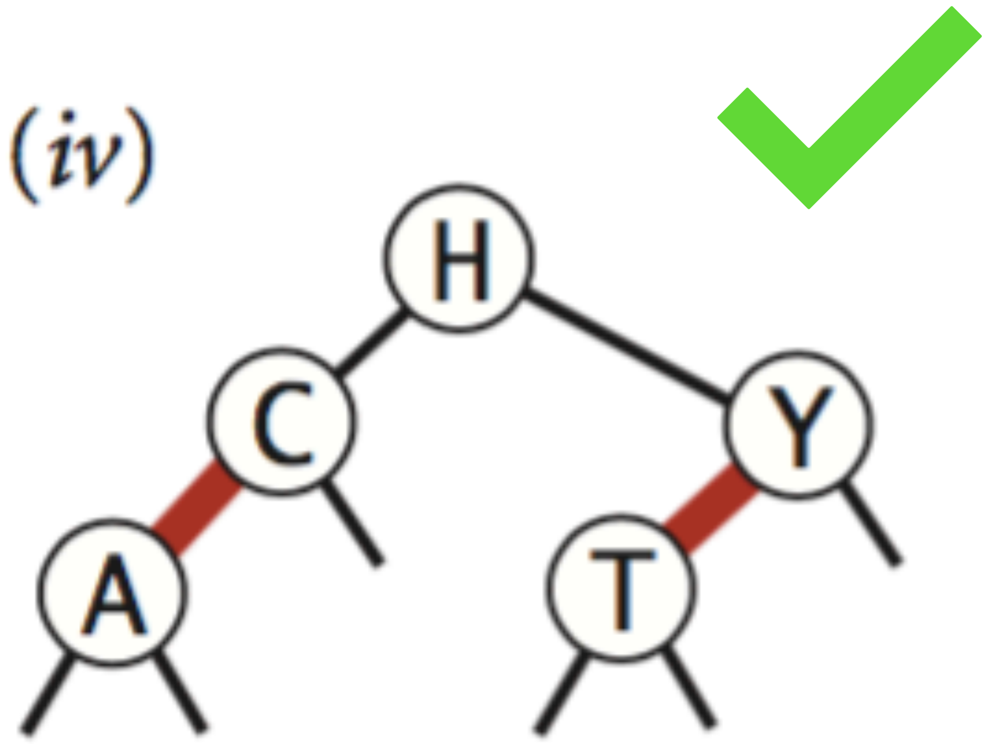
(ii)

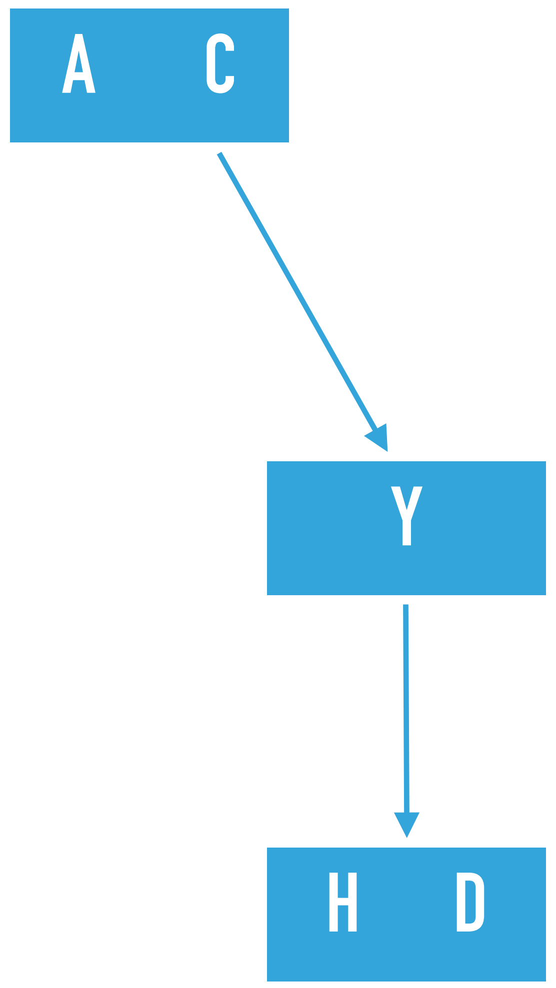


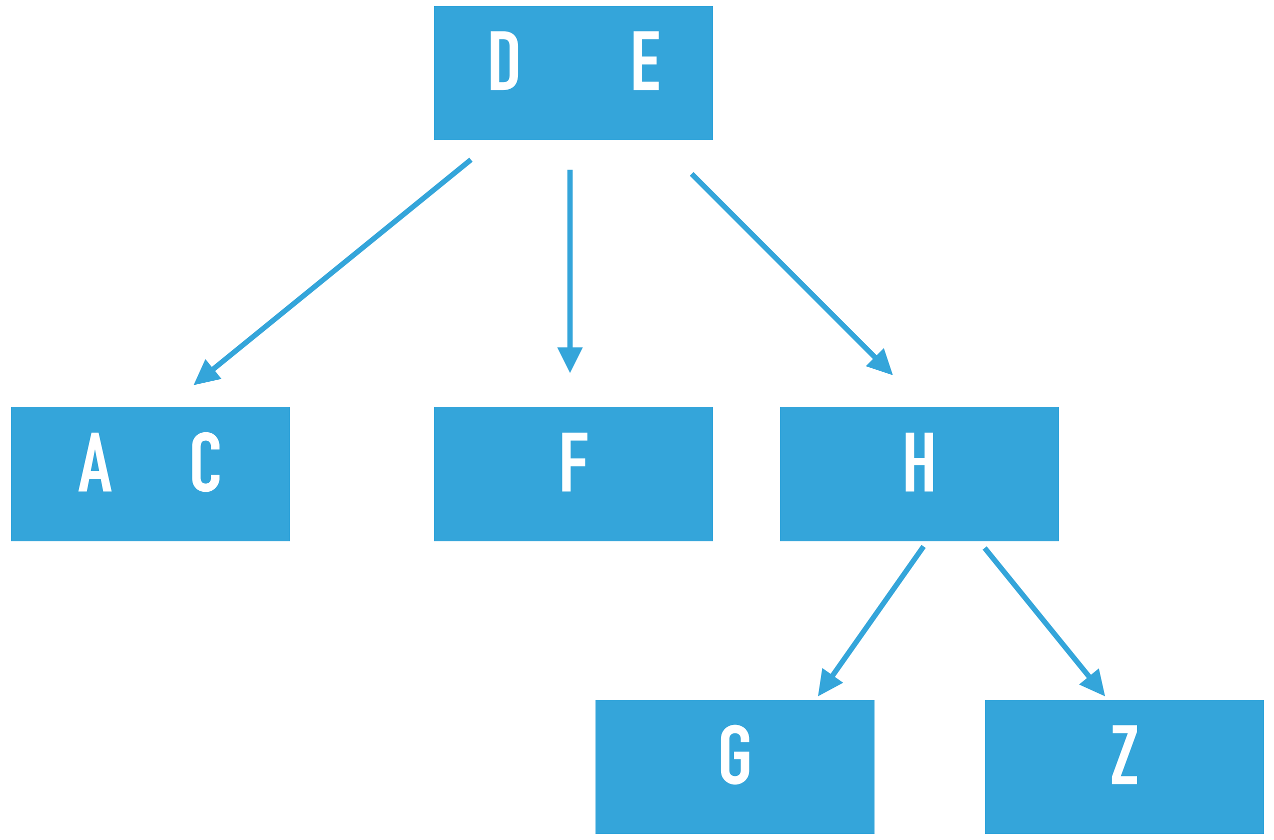
(iii)

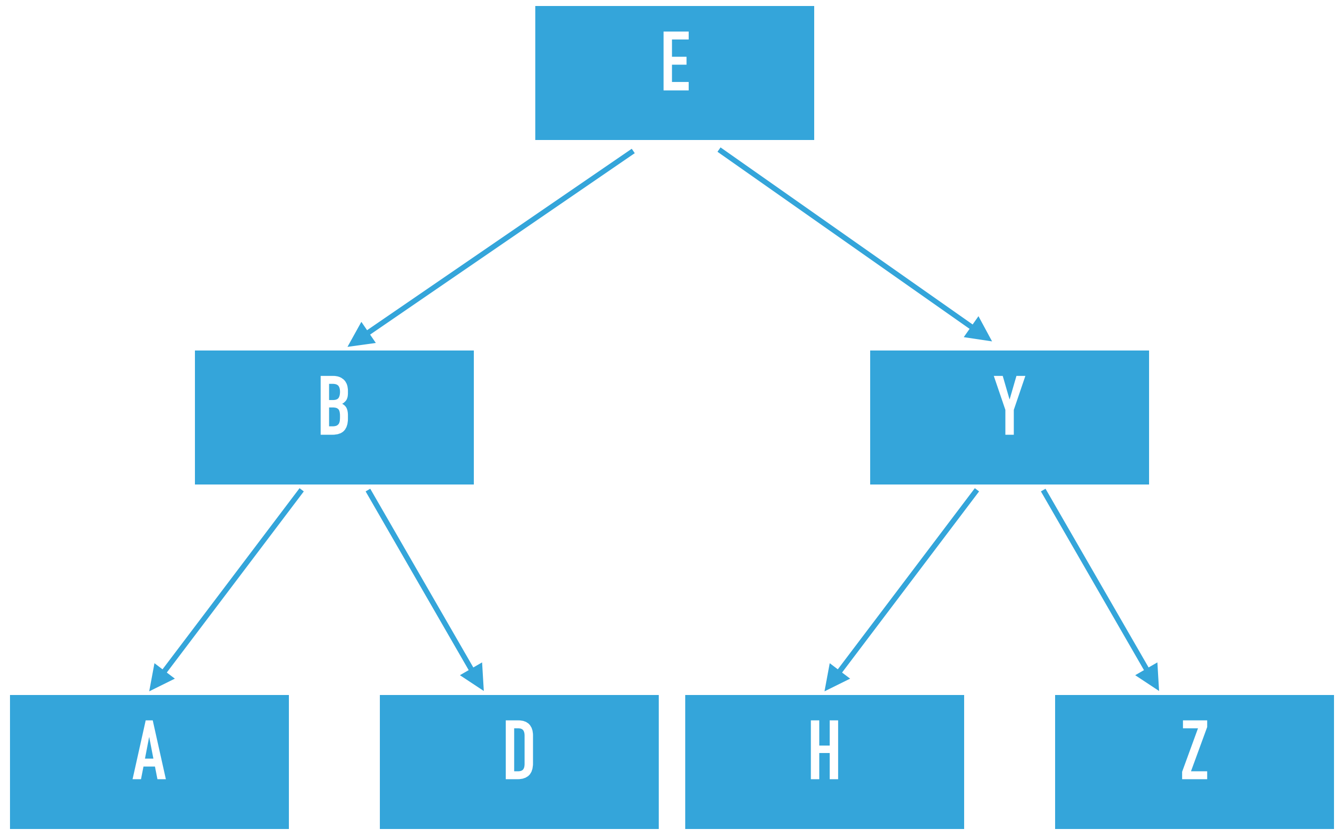


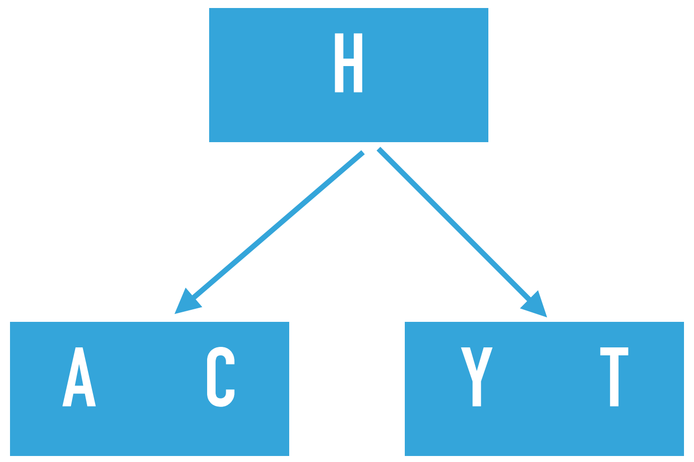
(iv)











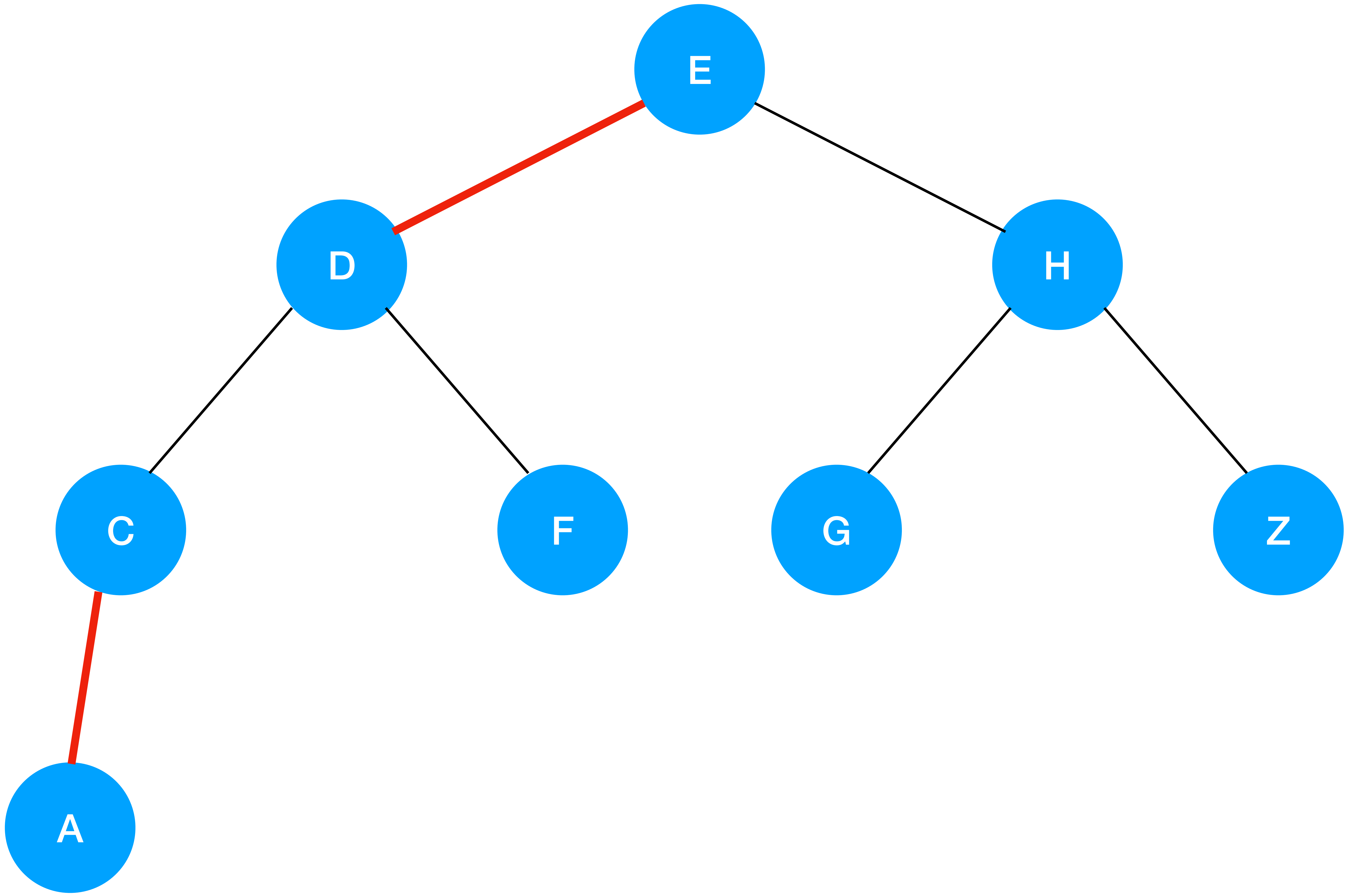
Question 3.1.7 IS RED BLACK

```
private boolean is23() { return is23(root); }

private boolean is23(Node x) {
    if (x == null)
        return true;

    //Est-ce que le lien à droite est rouge ?
    if (isRed(x.right))
        return false;

    //Est-ce que le noeud est connecté à deux liens rouges ?
    if (x != root && isRed(x) && isRed(x.left))
        return false;
    return is23(x.left) && is23(x.right);
}
```

Question 3.1.7 IS RED BLACK

```
private boolean isBalanced() {
        // nombre de liens noirs de la racine à la clé min
        int black = 0;
        Node x = root;
        while (x != null) {
            if (!isRed(x)) black++;
            x = x.left;
        }
        // Est-ce qu'on retrouve ce nombre de liens pour chaque feuille ?
        return isBalanced(root, black);
    }


private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

Question 3.1.7 IS RED BLACK

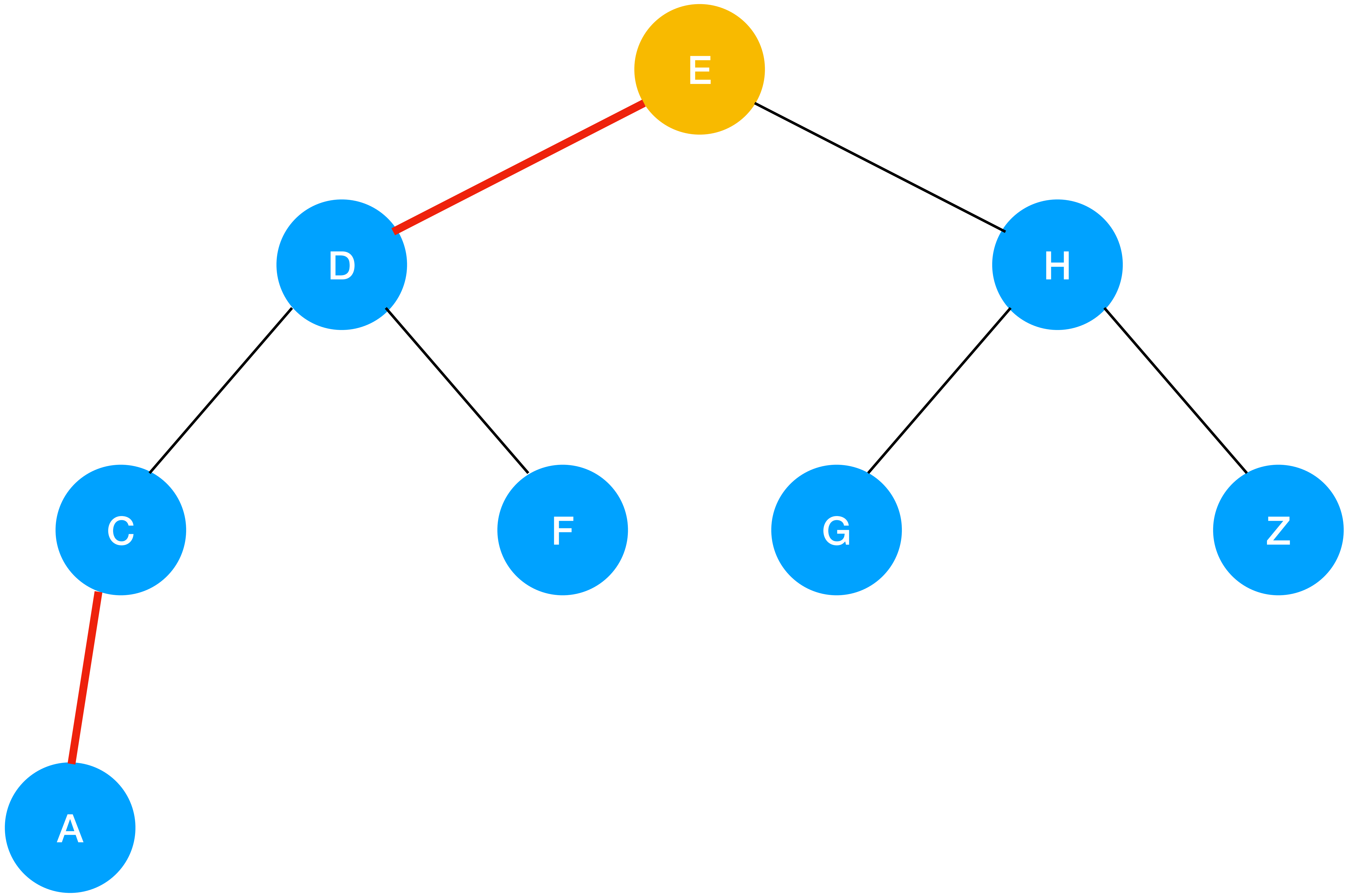


Node =

Black = 0

```
int black = 0;
Node x = root;
```

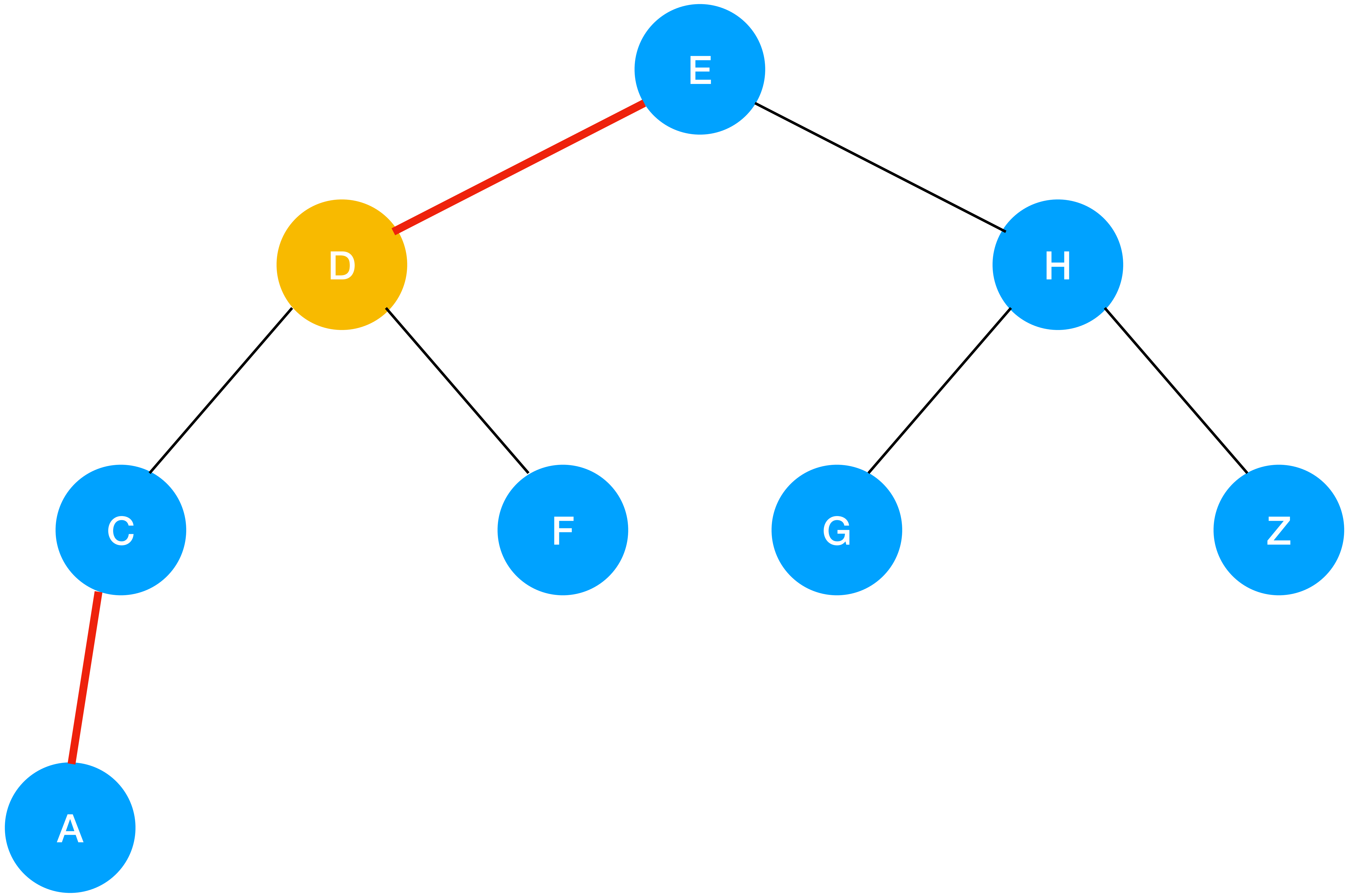
Question 3.1.7 IS RED BLACK



Node = E
Black = 1

```
while (x != null) {
        if (!isRed(x)) black++;
        x = x.left;
}
```

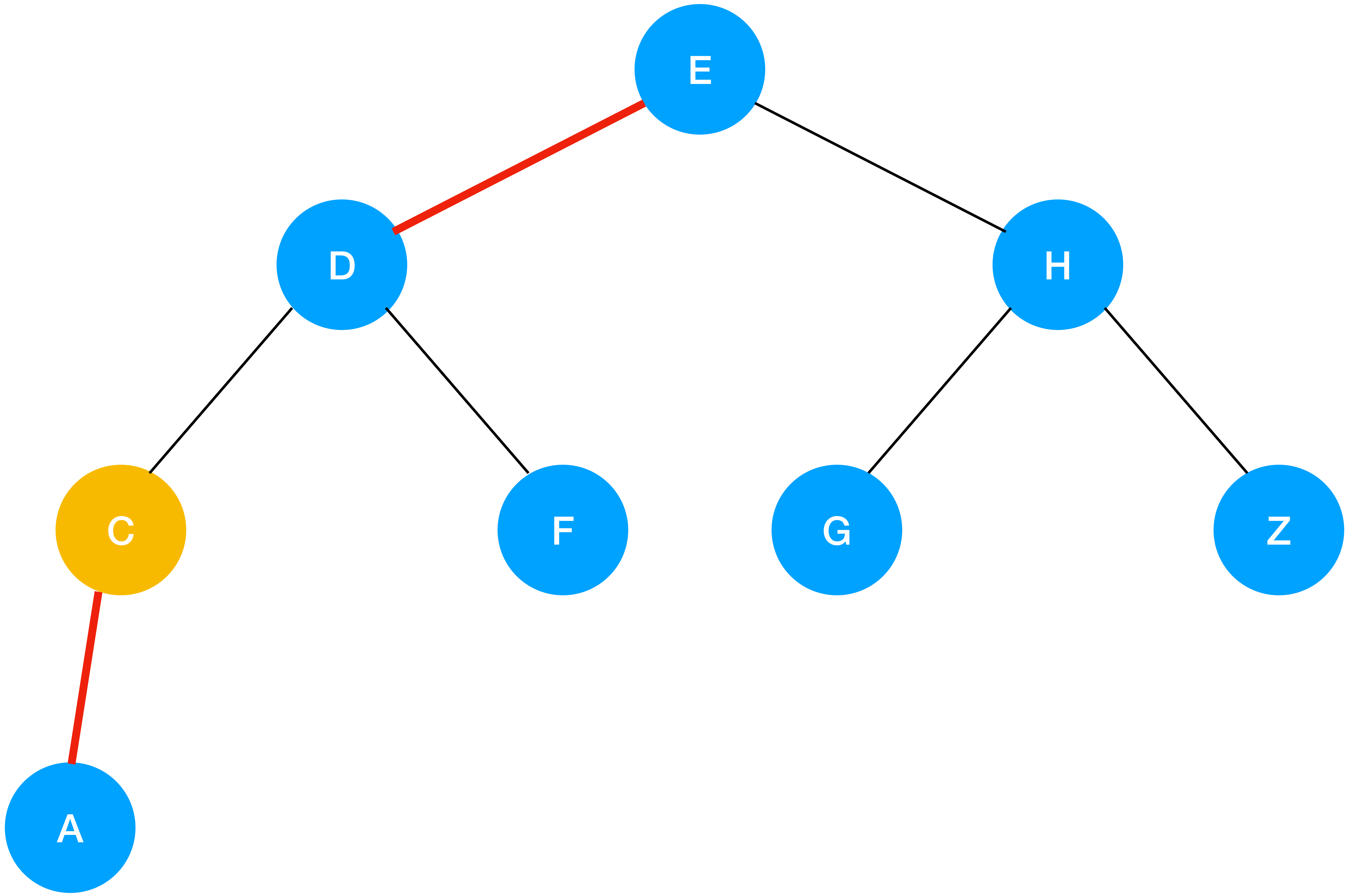
Question 3.1.7 IS RED BLACK



Node = D
Black = 1

```
while (x != null) {
        if (!isRed(x)) black++;
        x = x.left;
}
```

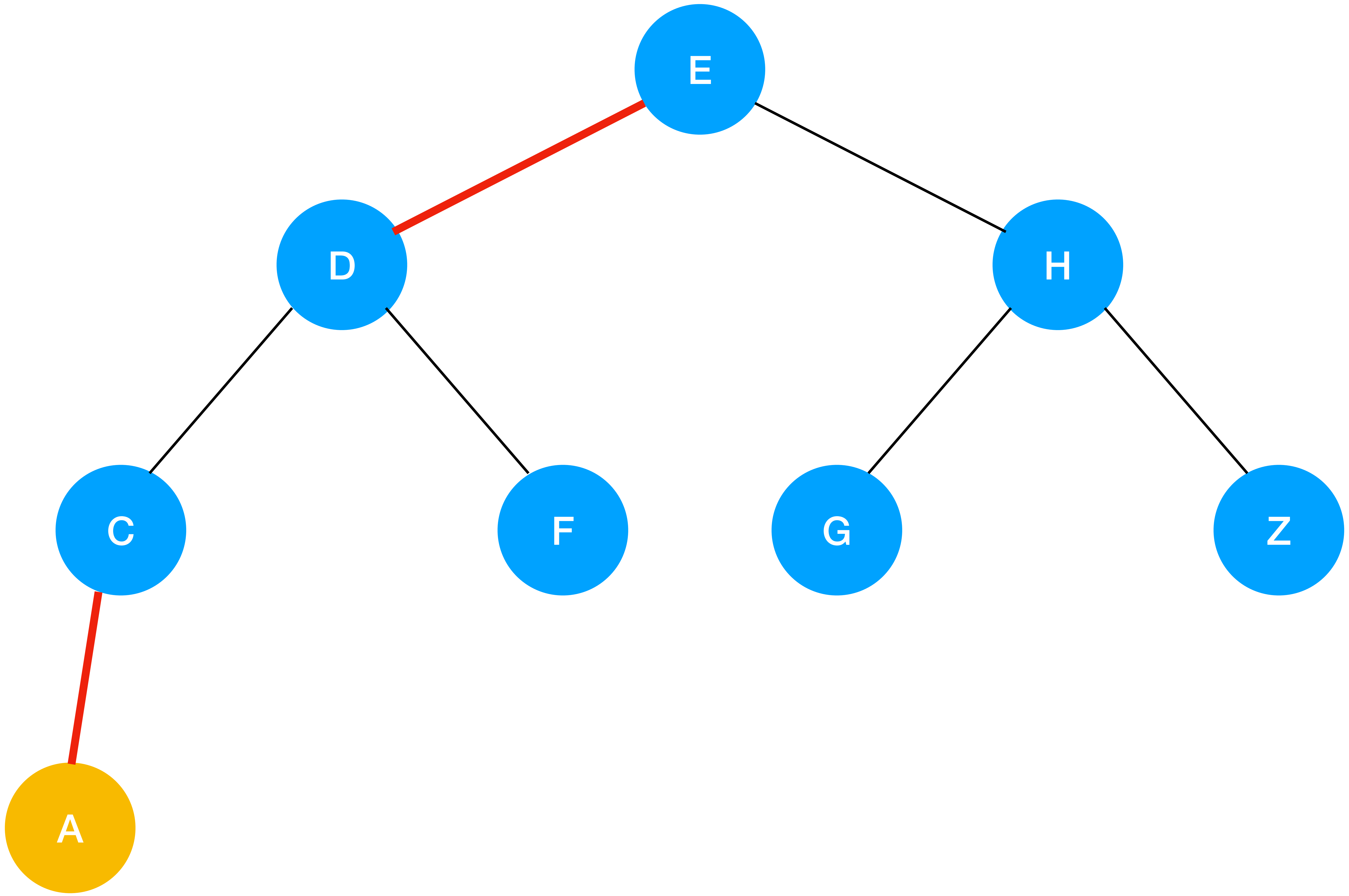
Question 3.1.7 IS RED BLACK



Node = C
Black = 2

```
while (x != null) {
        if (!isRed(x)) black++;
        x = x.left;
}
```

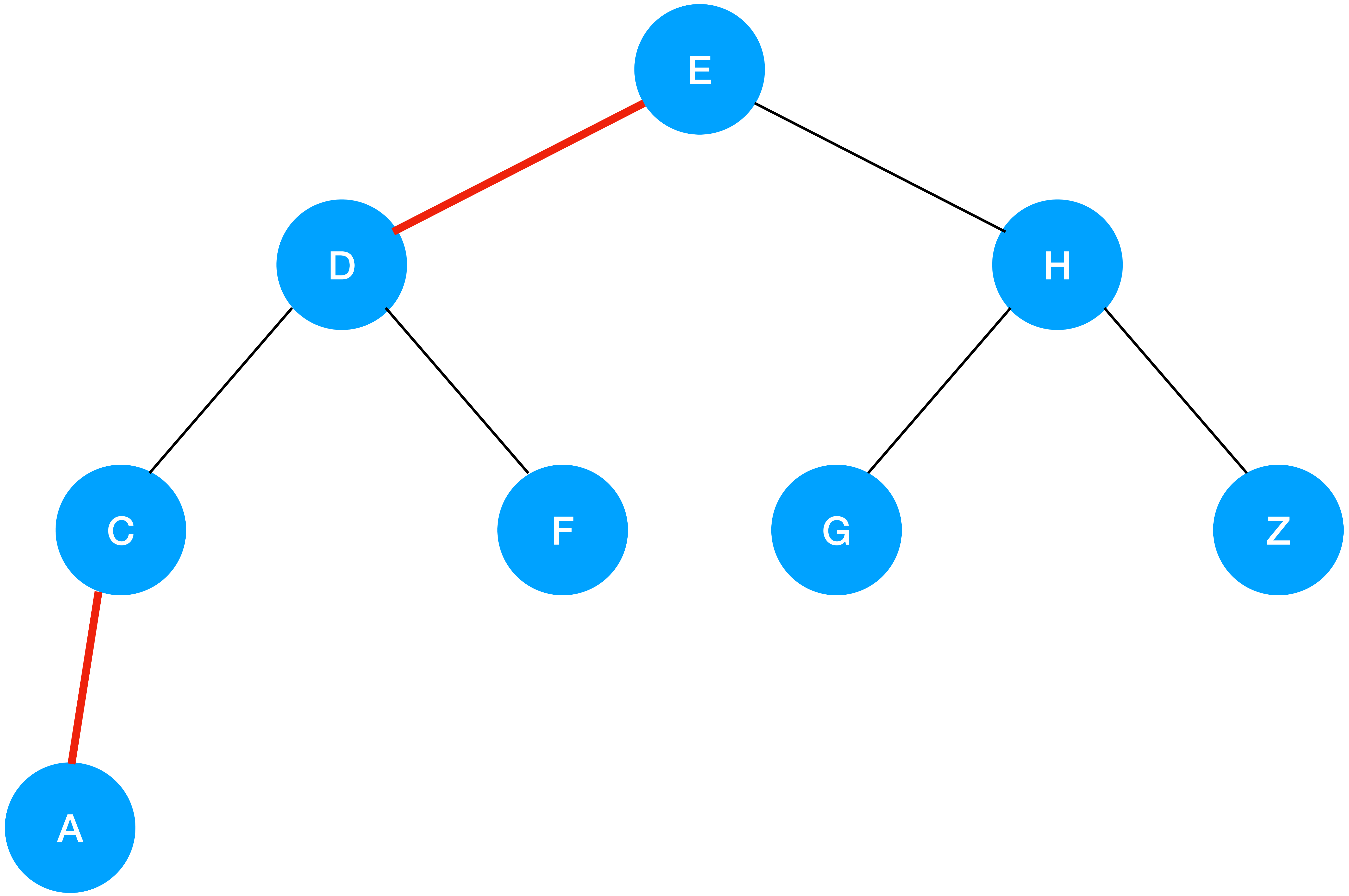
Question 3.1.7 IS RED BLACK



Node = A
Black = 2

```
while (x != null) {
        if (!isRed(x)) black++;
        x = x.left;
}
```

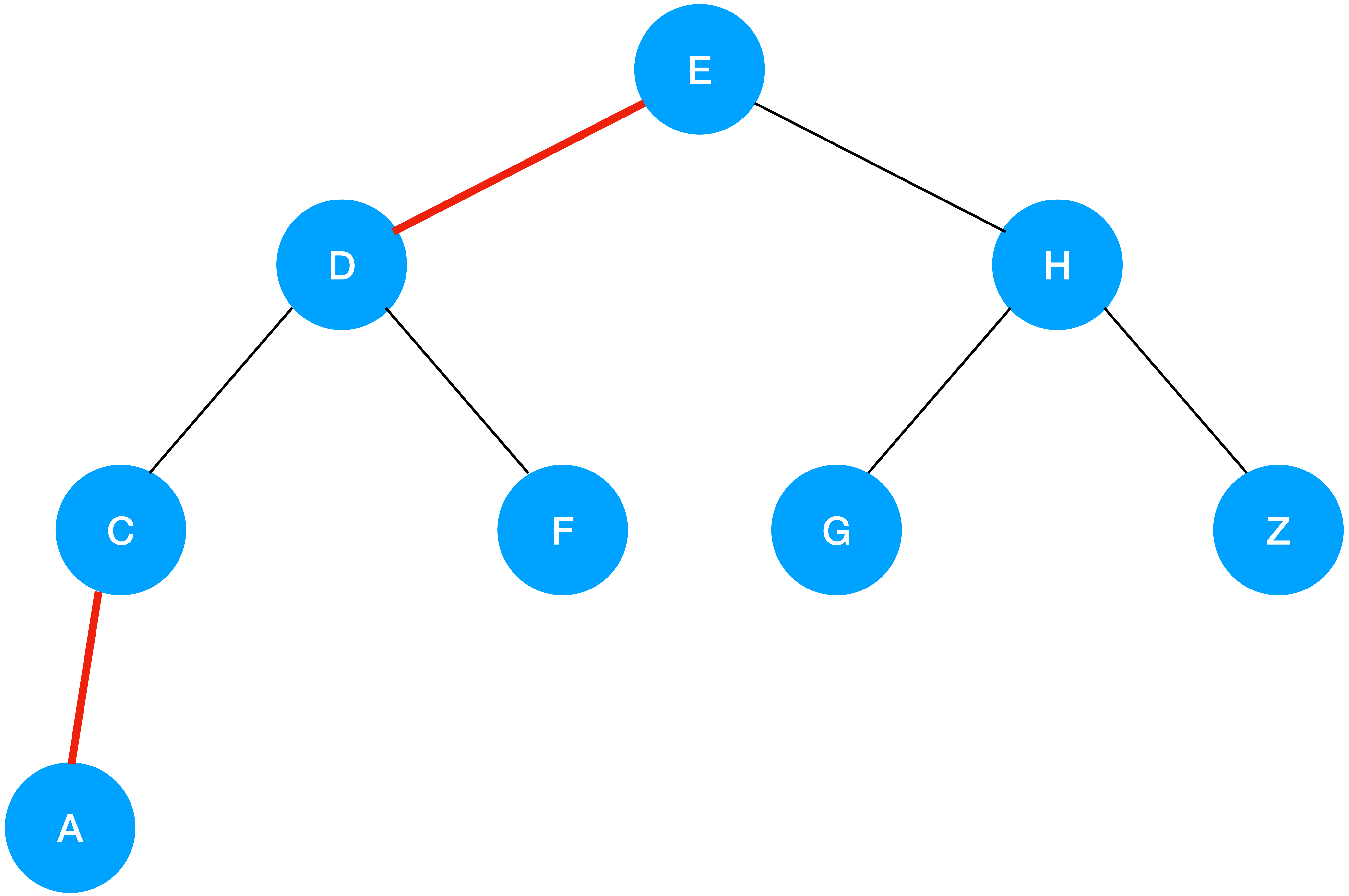
Question 3.1.7 IS RED BLACK



Node = null
Black = 2

```
while (x != null) {
        if (!isRed(x)) black++;
        x = x.left;
}
```

Question 3.1.7 IS RED BLACK

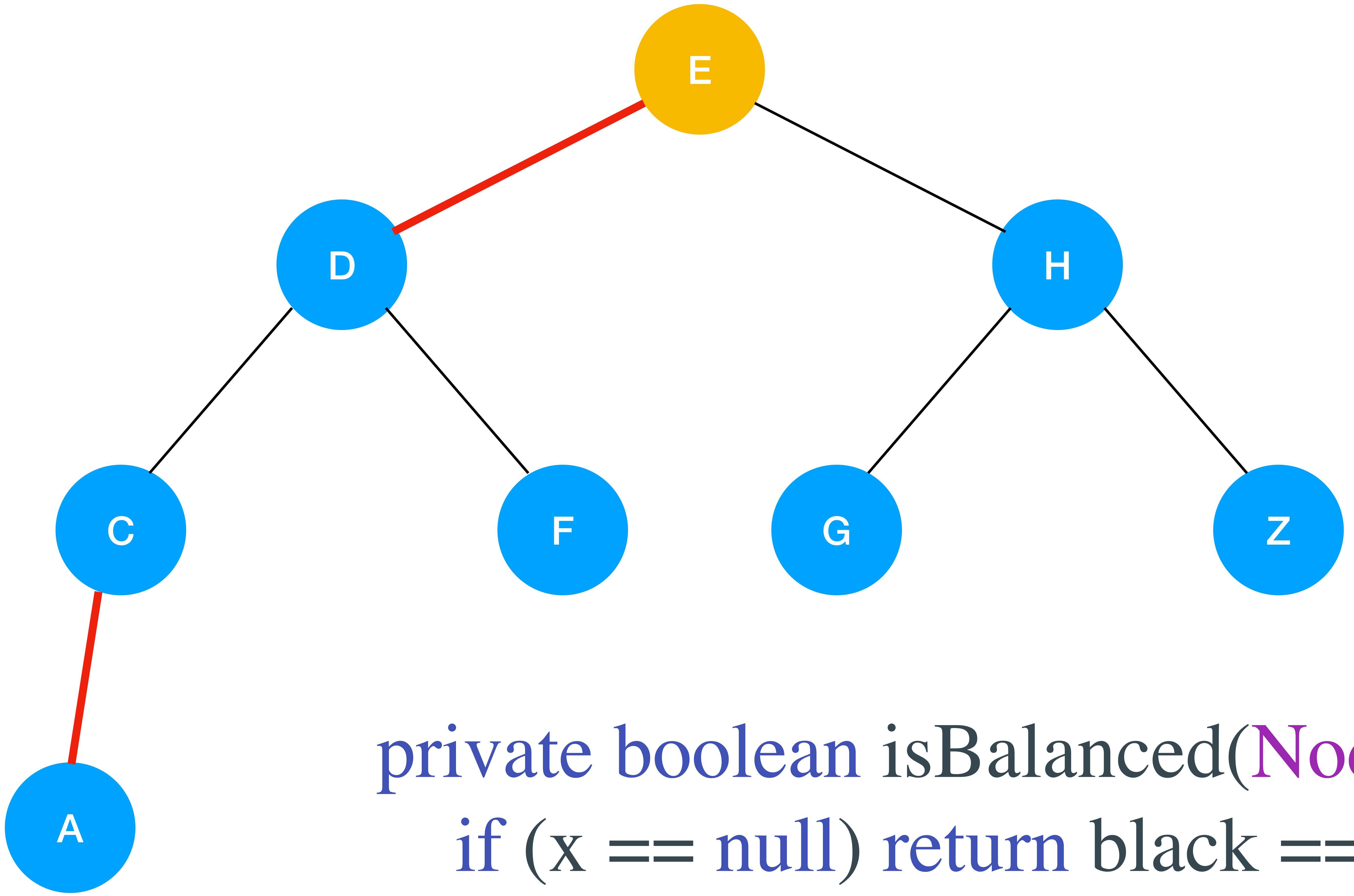


Node =

Black = 2

```
return isBalanced(root, black);
```

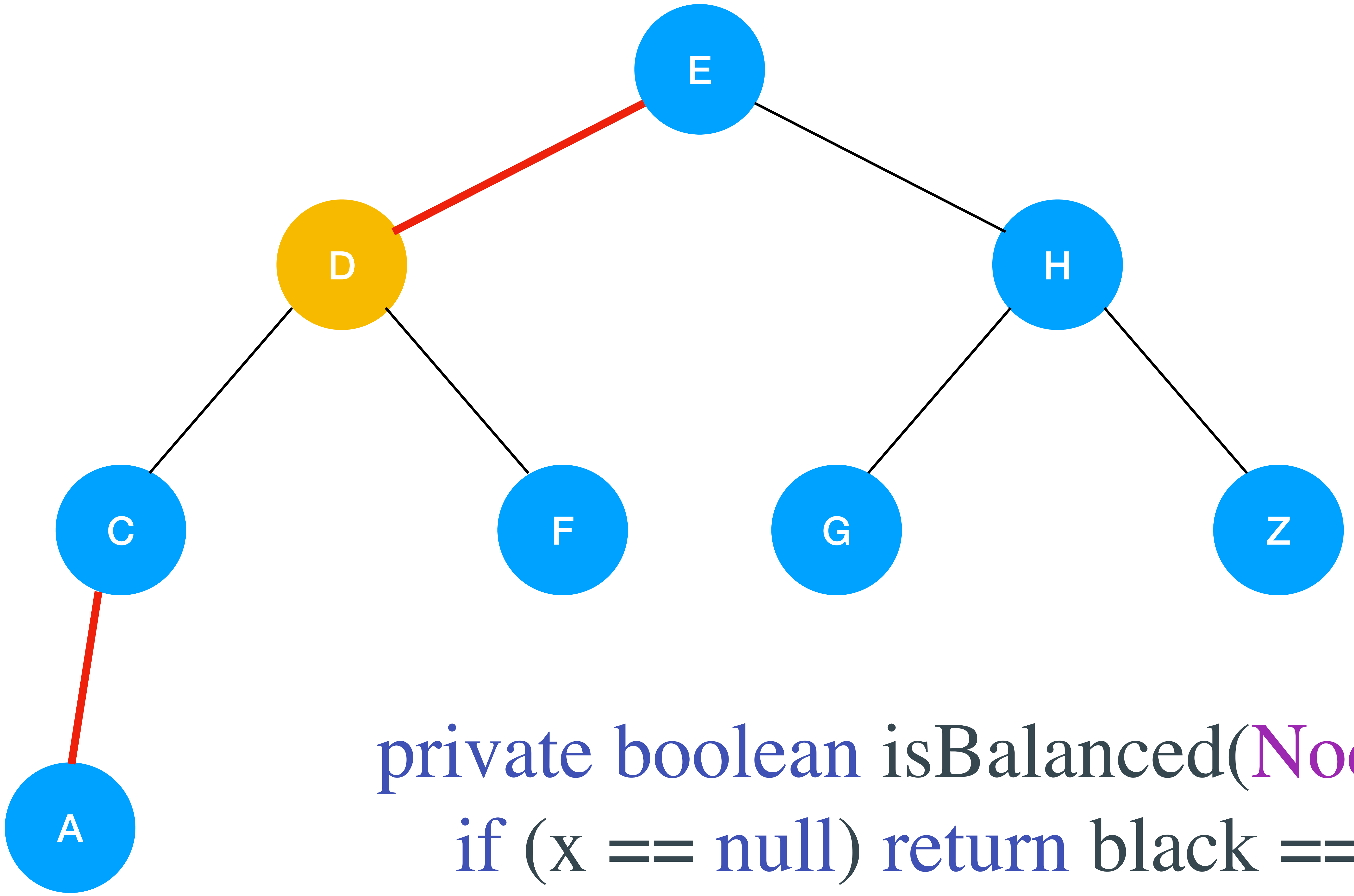
Question 3.1.7 IS RED BLACK



Node = E
Black = 1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

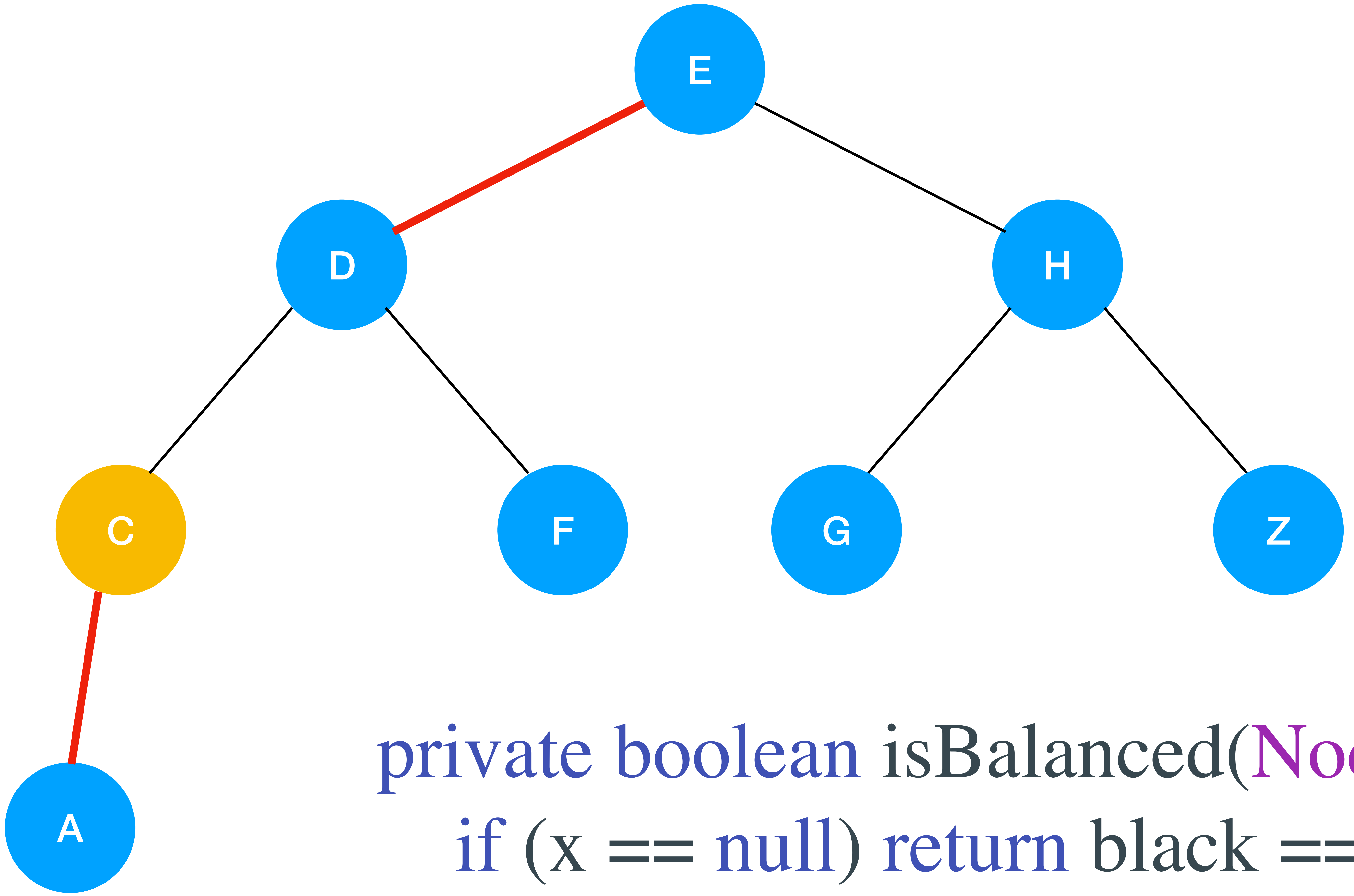
Question 3.1.7 IS RED BLACK



Node = D
Black = 1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

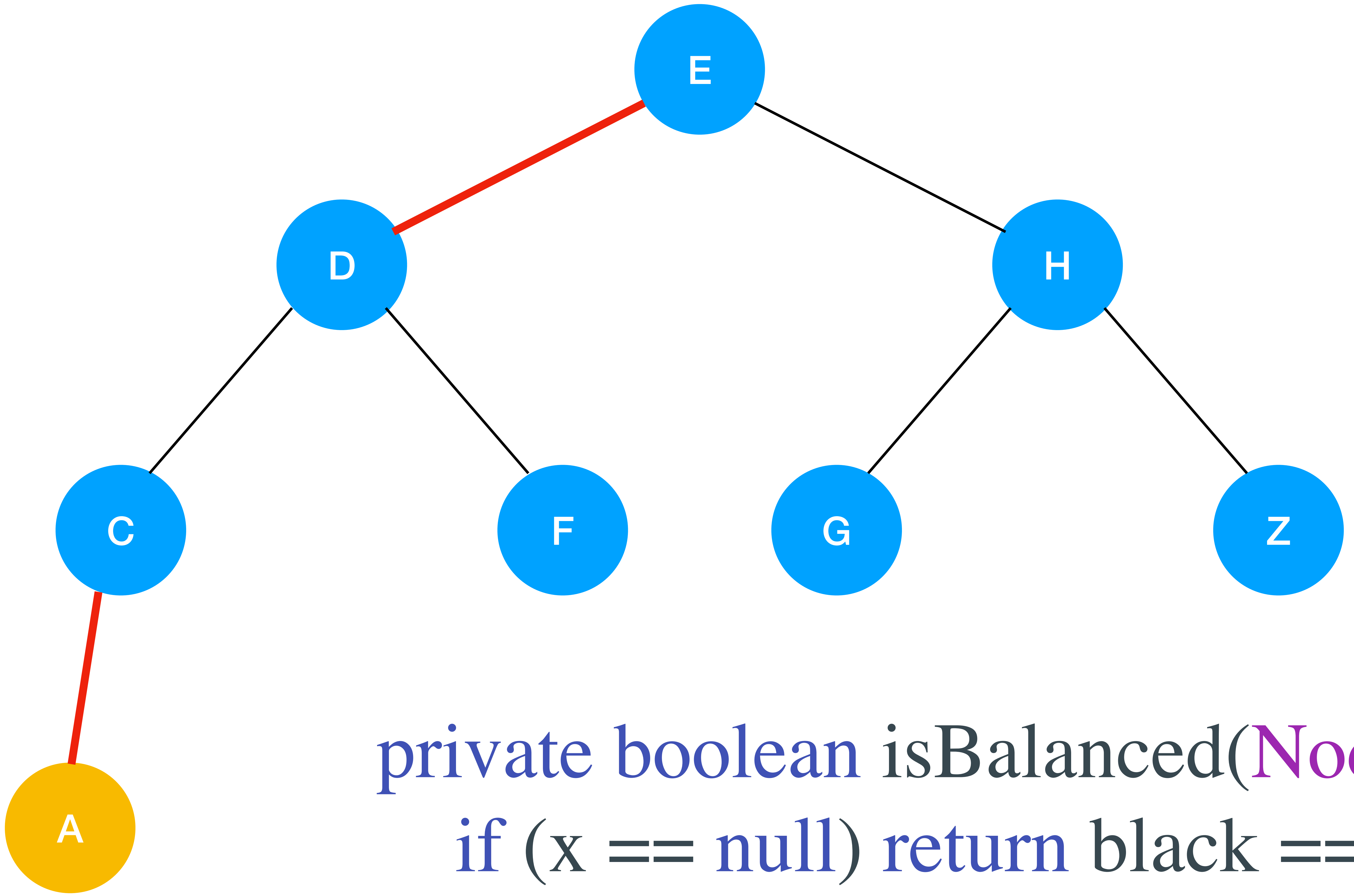
Question 3.1.7 IS RED BLACK



Node = C
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

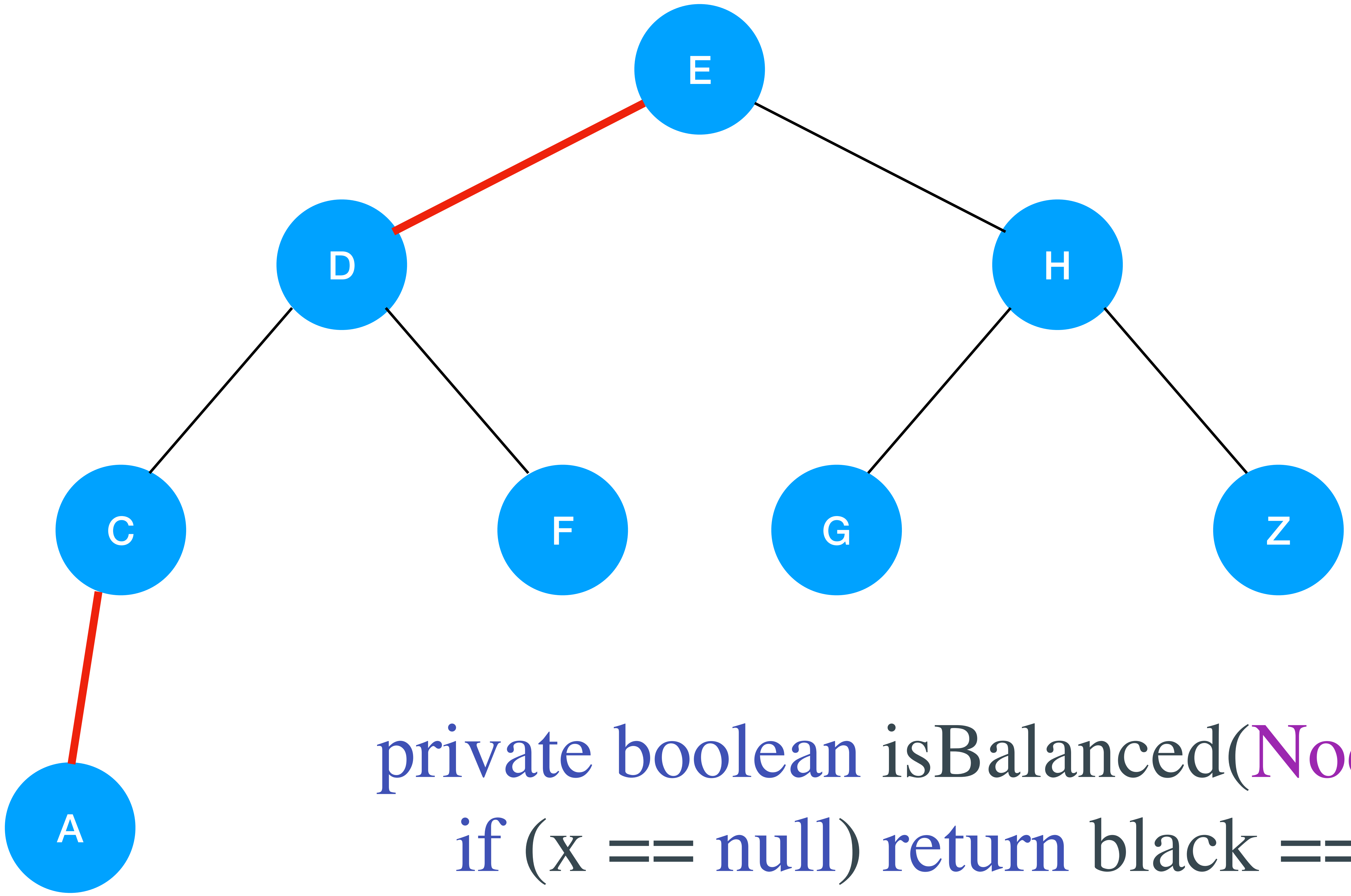
Question 3.1.7 IS RED BLACK



Node = A
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

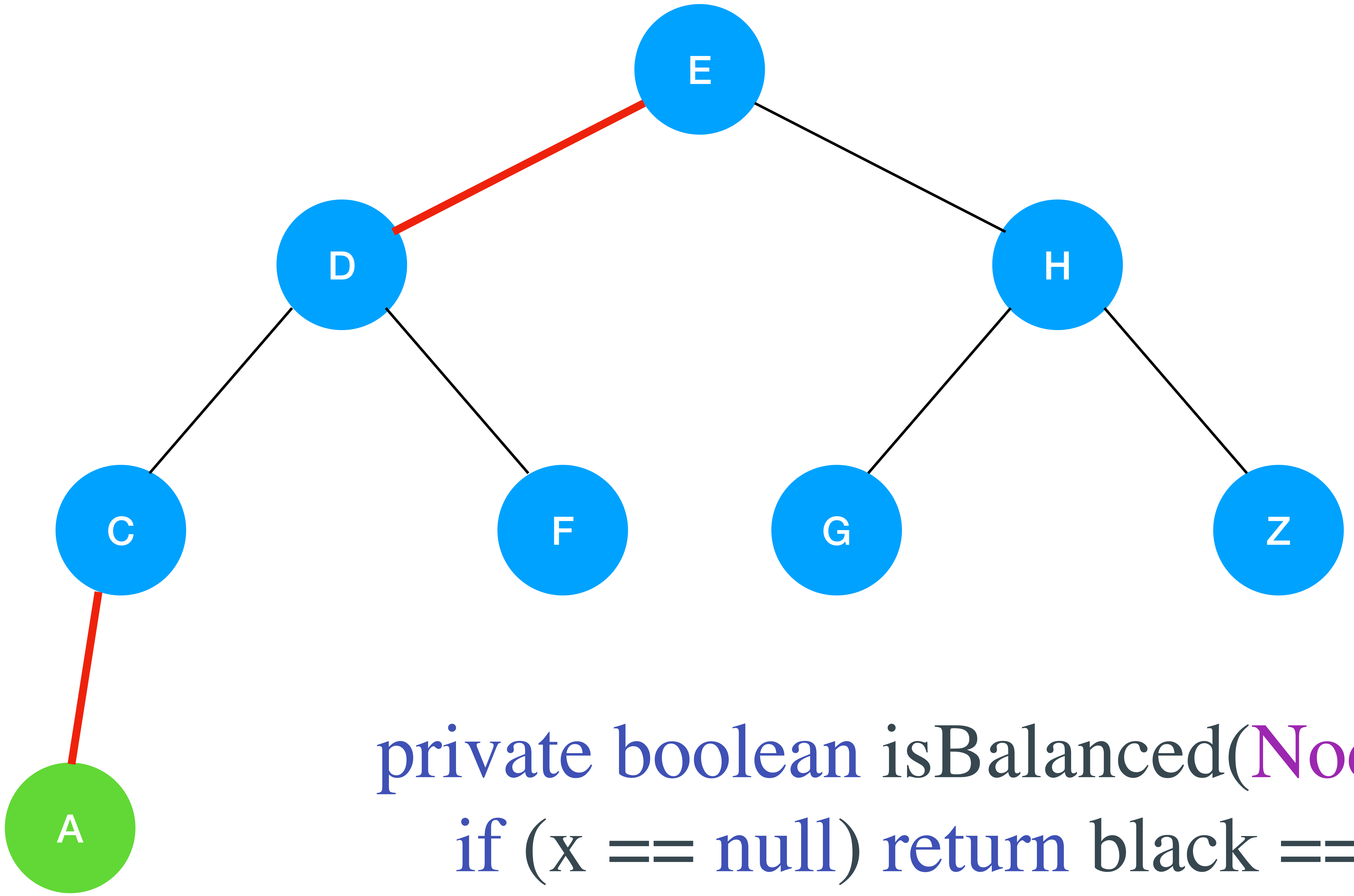
Question 3.1.7 IS RED BLACK



Node = null
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

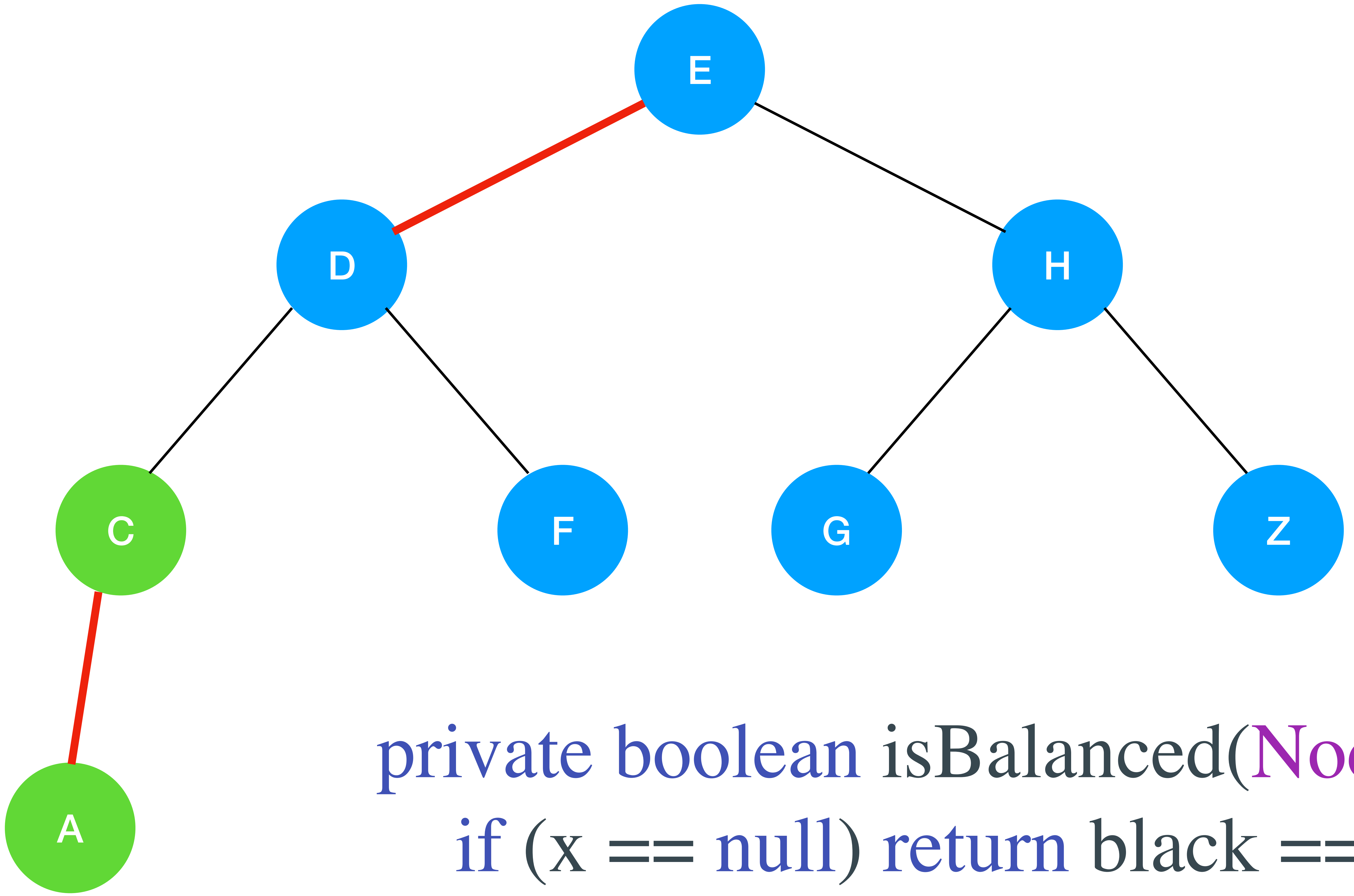
Question 3.1.7 IS RED BLACK



Node = A
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

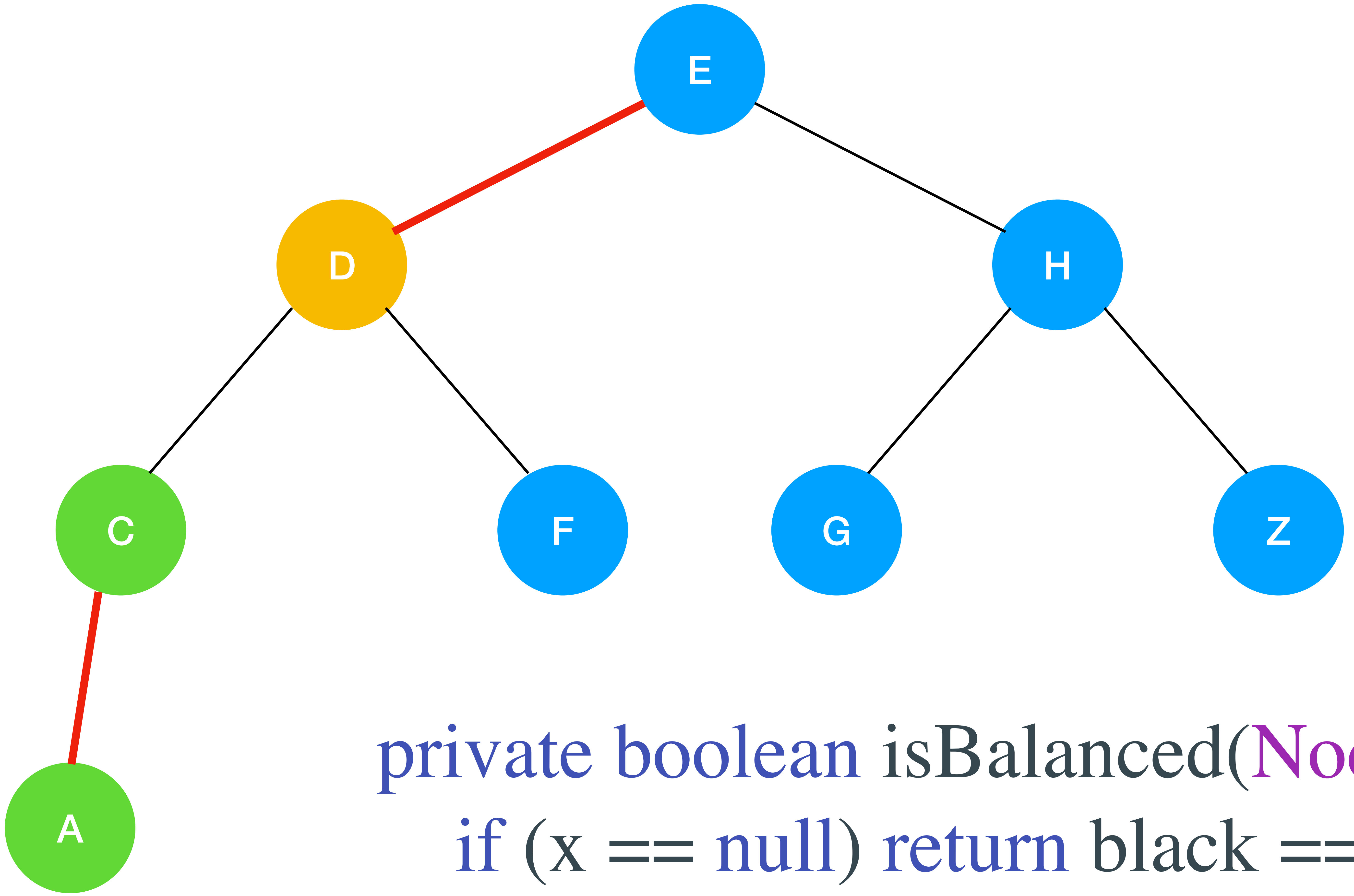
Question 3.1.7 IS RED BLACK



Node = C
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

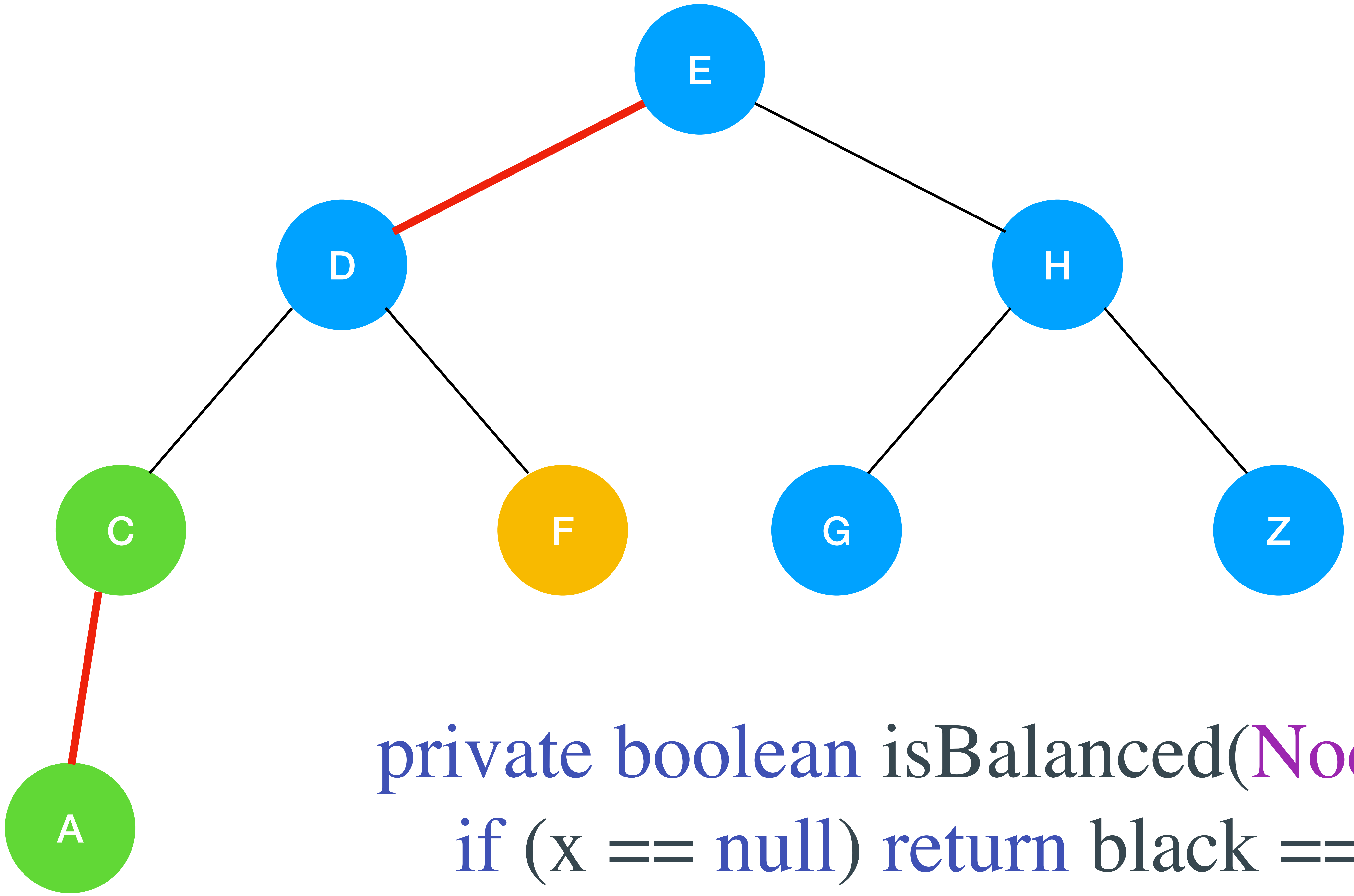
Question 3.1.7 IS RED BLACK



Node = D
Black = 1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

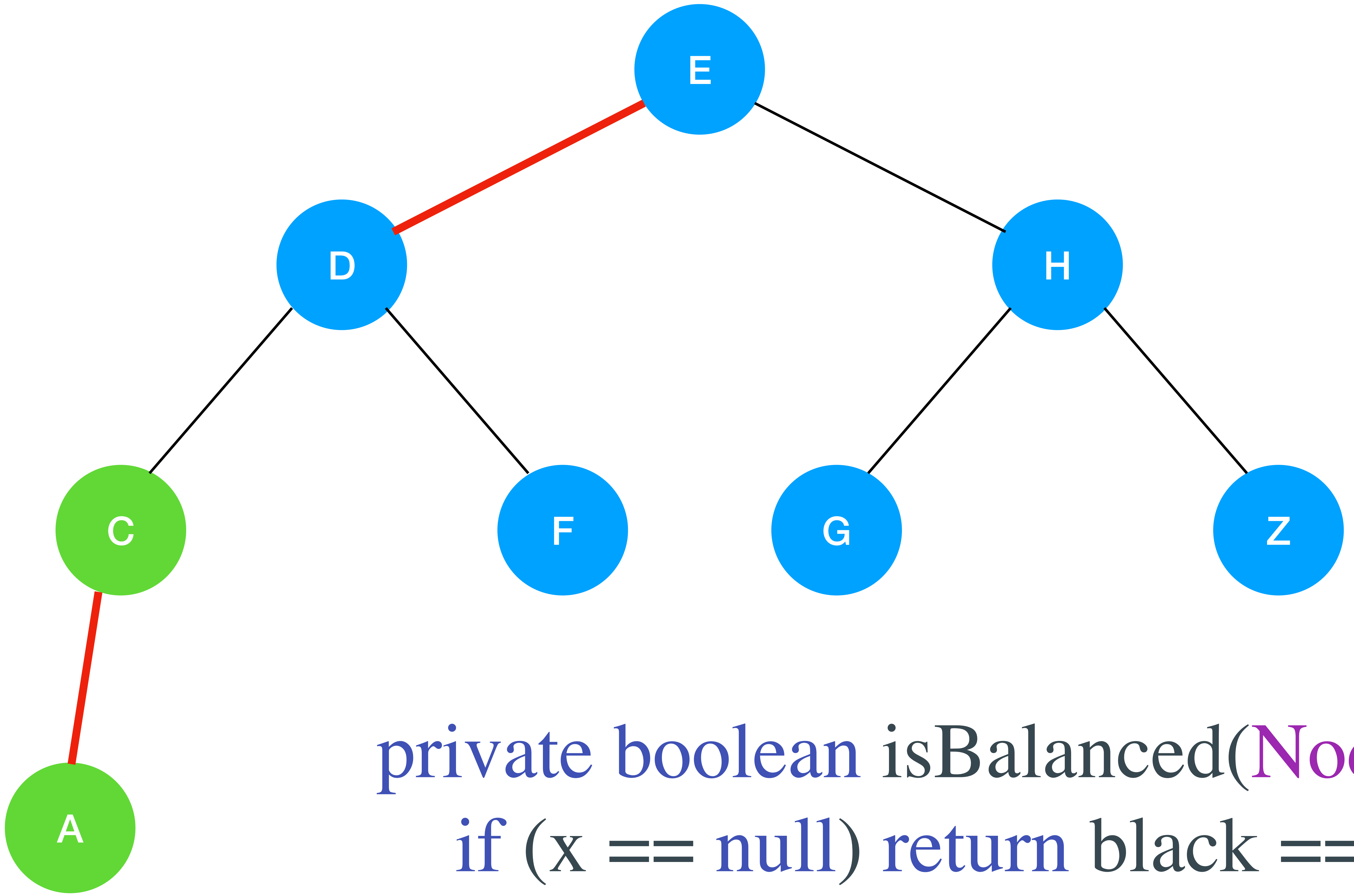
Question 3.1.7 IS RED BLACK



Node = F
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

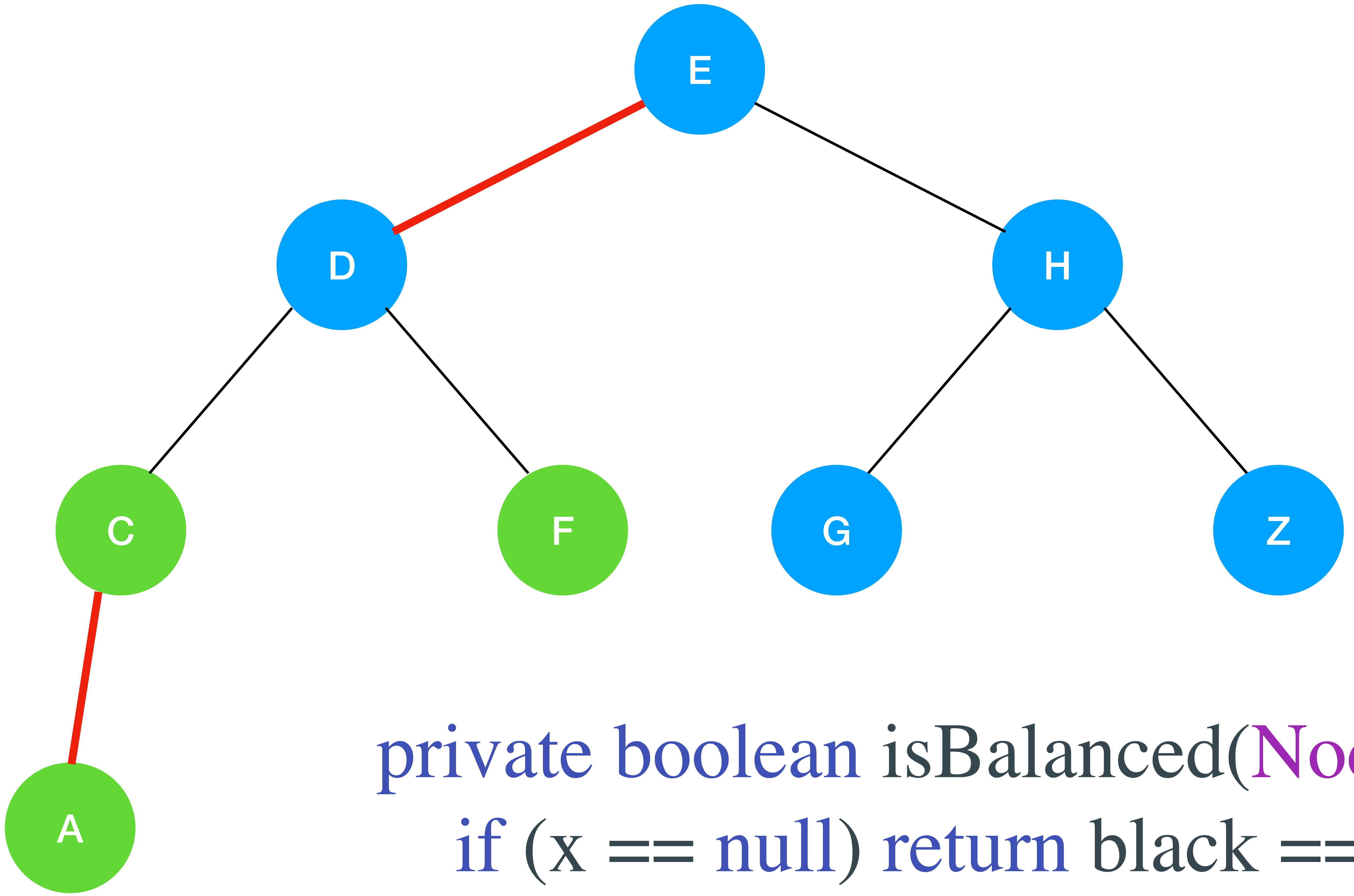
Question 3.1.7 IS RED BLACK



Node = null
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

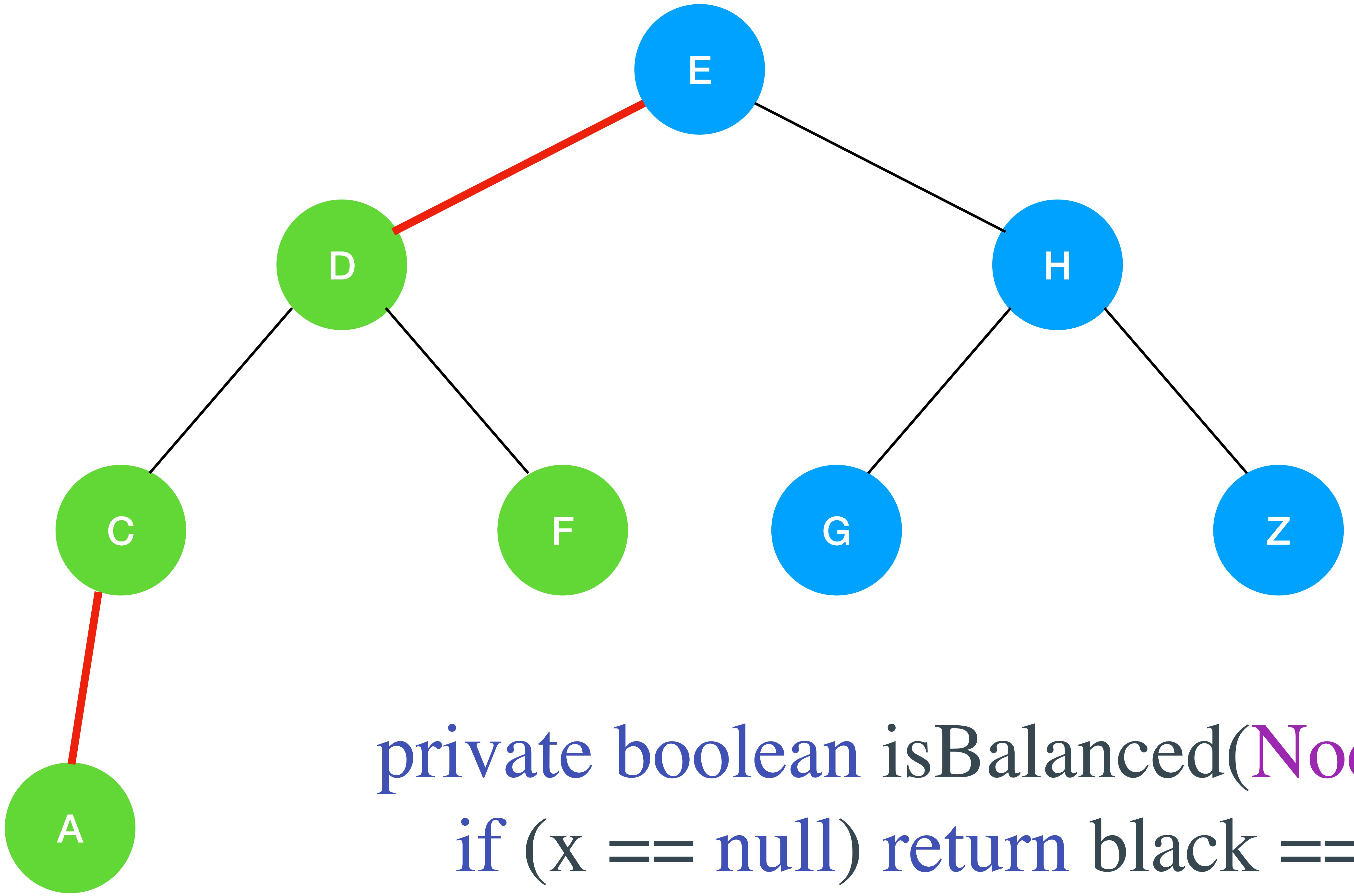
Question 3.1.7 IS RED BLACK



Node = F
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

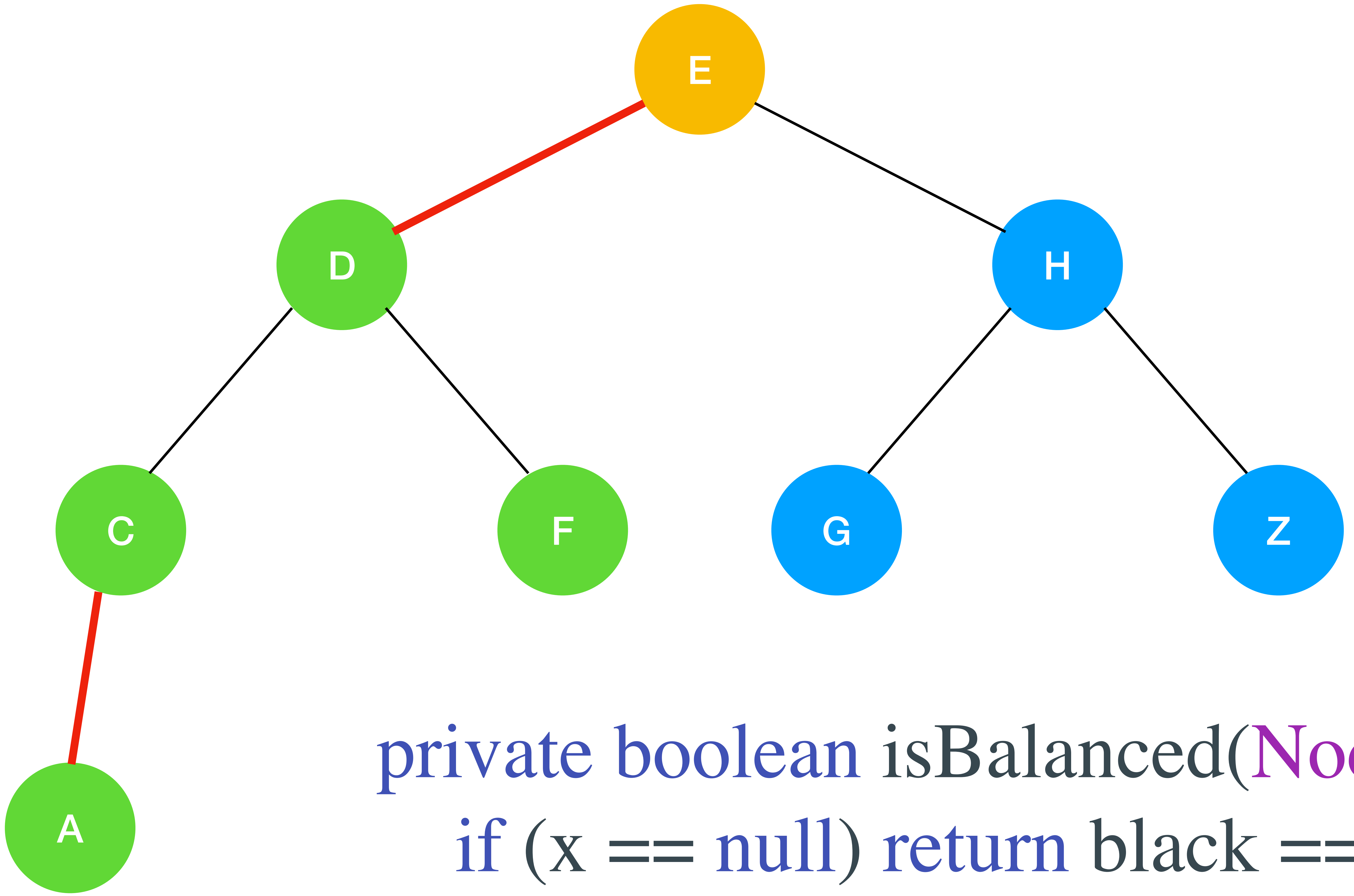
Question 3.1.7 IS RED BLACK



Node = D
Black = 1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

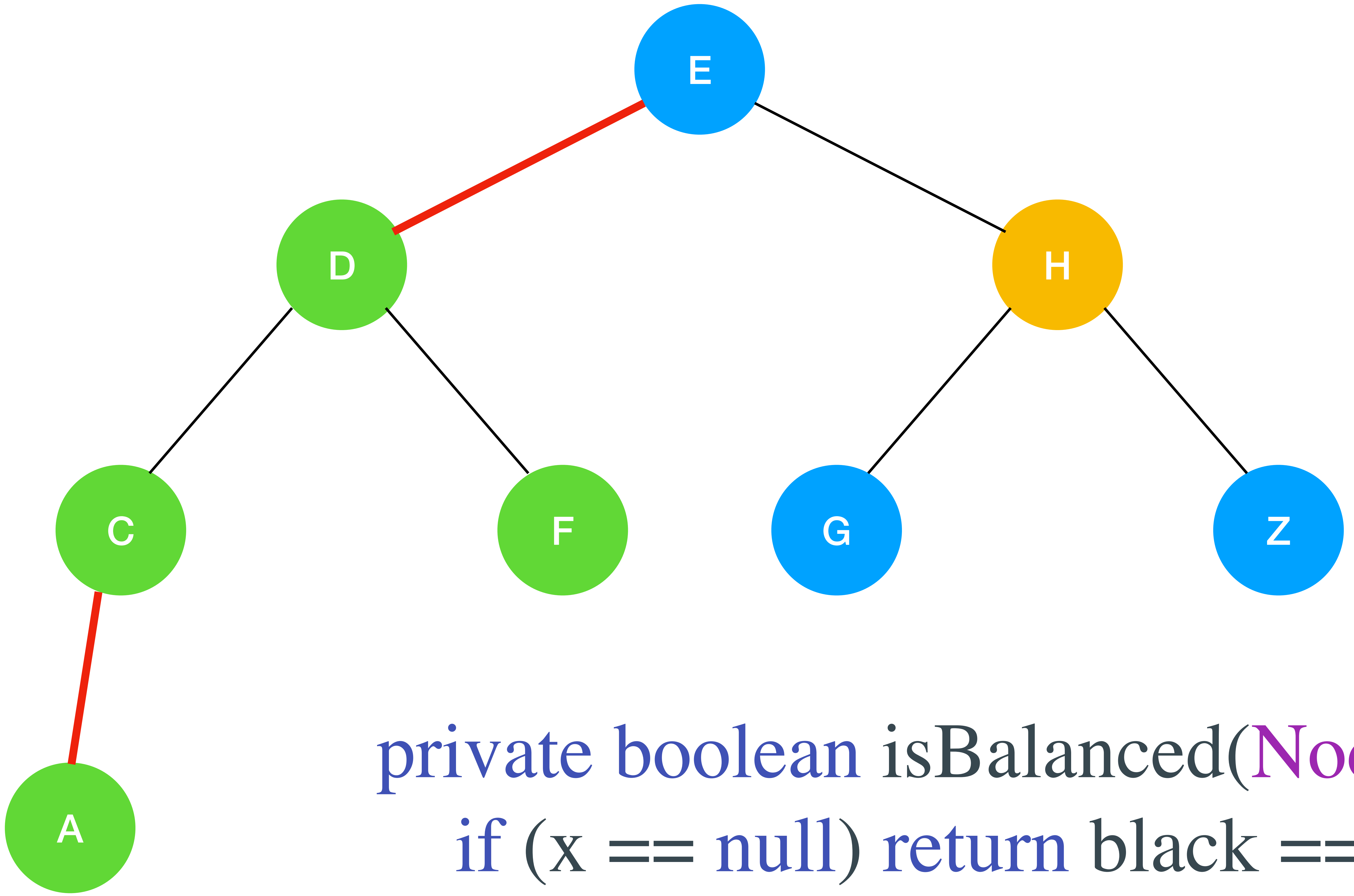
Question 3.1.7 IS RED BLACK



Node = E
Black = 1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

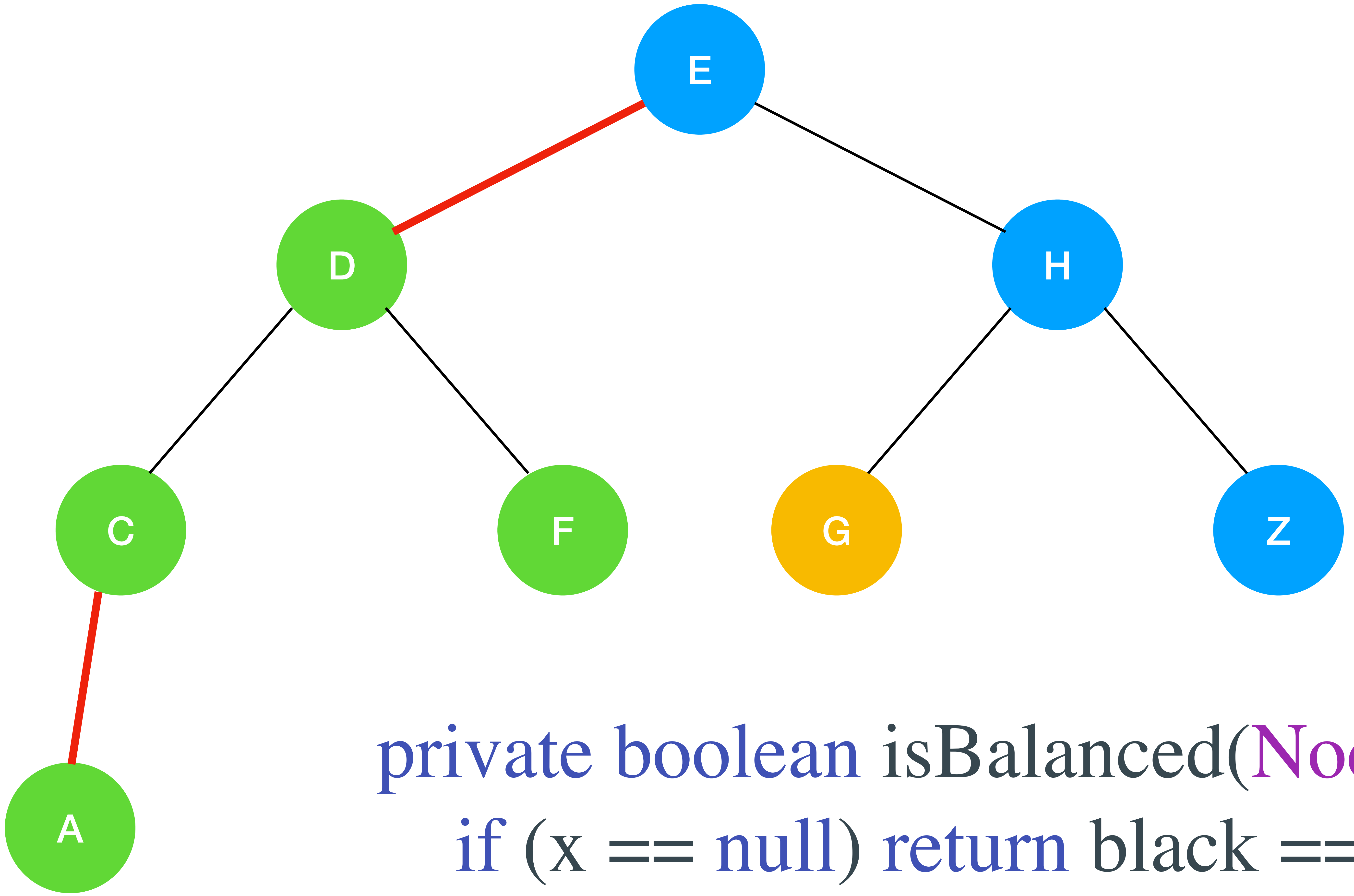
Question 3.1.7 IS RED BLACK



Node = H
Black = 0

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

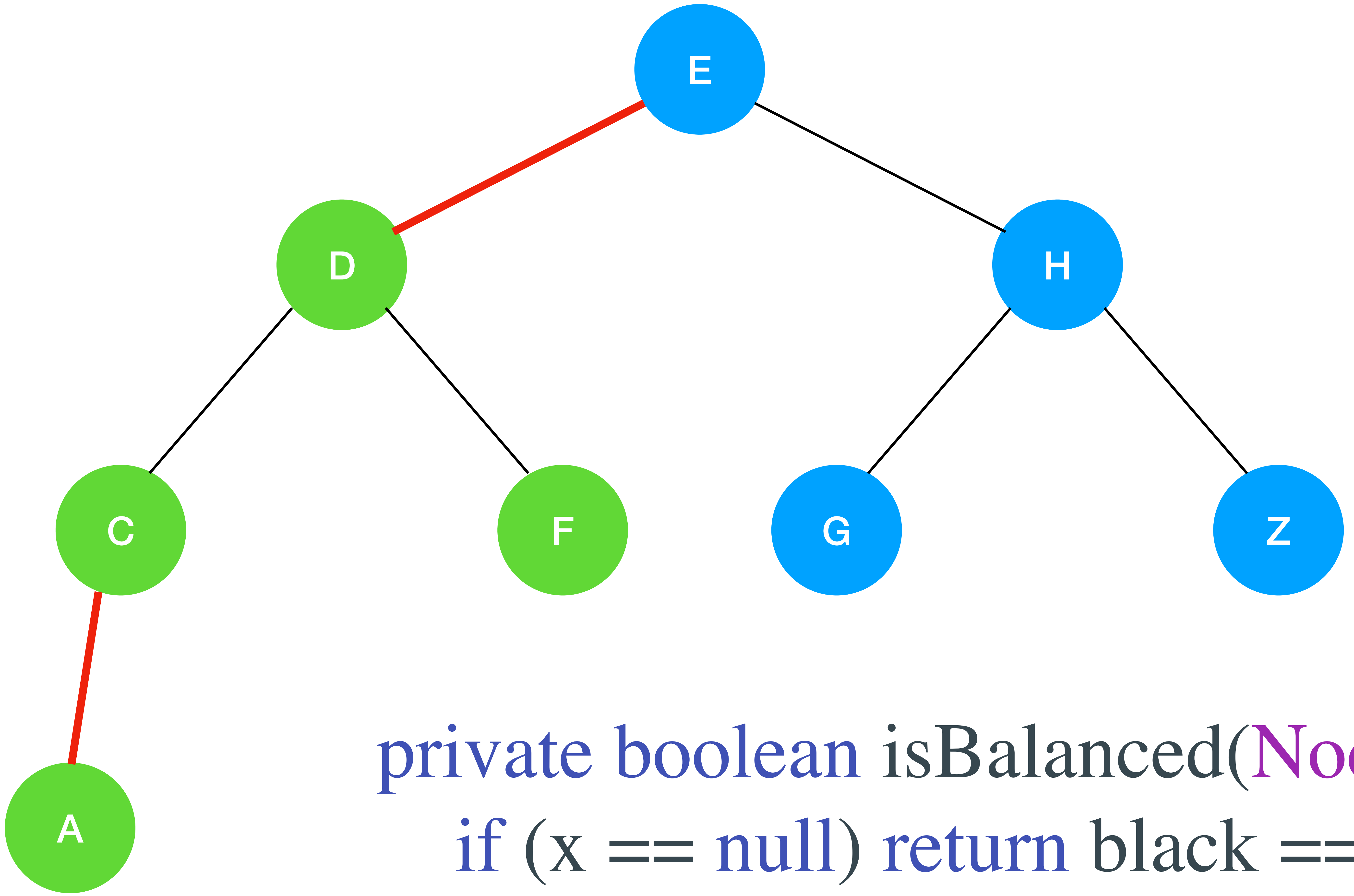
Question 3.1.7 IS RED BLACK



Node = G
Black = -1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

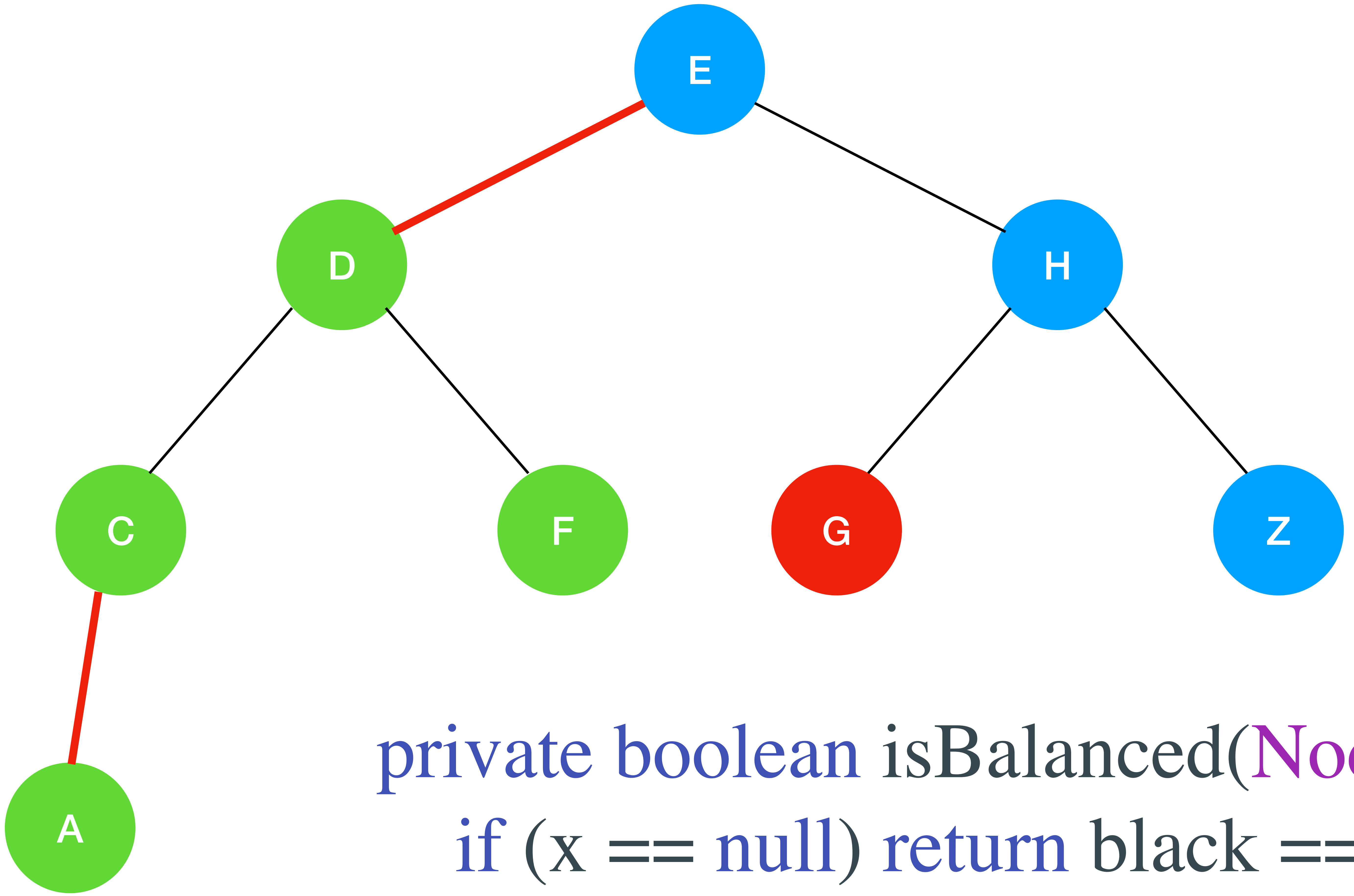
Question 3.1.7 IS RED BLACK



Node = null
Black = -1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

Question 3.1.7 IS RED BLACK



Node = G
Black = -1

```
private boolean isBalanced(Node x, int black) {
    if (x == null) return black == 0;
    if (!isRed(x)) black--;
    return isBalanced(x.left, black) && isBalanced(x.right, black);
}
```

algorithm (data structure)	worst-case cost (after N inserts)		average-case cost (after N random inserts)		efficiently support ordered operations?
	search	insert	search hit	insert	
sequential search (unordered linked list)	N	N	$N/2$	N	no
binary search (ordered array)	$\lg N$	N	$\lg N$	$N/2$	yes
binary tree search (BST)	N	N	$1.39 \lg N$	$1.39 \lg N$	yes
2-3 tree search (red-black BST)	$2 \lg N$	$2 \lg N$	$1.00 \lg N$	$1.00 \lg N$	yes

Cost summary for symbol-table implementations (updated)